Dutch vs. First-Price Auctions with Expectations-Based Loss-Averse Bidders*

Benjamin Balzer[†] Antonio Rosato[‡] Jonas von Wangenheim[§]

February 22, 2022

Abstract

We study Dutch and first-price auctions with expectations-based loss-averse bidders and show that the strategic equivalence between these formats no longer holds. Intuitively, as the Dutch auction unfolds, a bidder becomes more optimistic about her chances of winning; this stronger "attachment" effect pushes her to bid more aggressively than in the first-price auction. Thus, Dutch auctions raise more revenue than first-price ones. Indeed, the Dutch auction raises the most revenue among standard auction formats. Our results imply that dynamic mechanisms that make bidders more optimistic raise more revenue, thereby rationalizing the use of descending-price mechanisms by sellers in the field.

JEL classification: D44, D81, D82.

Keywords: Loss Aversion; Dutch Auctions; Revenue Equivalence; Personal Equilibrium.

---

*We are grateful to the editor (Marzena Rostek) and two anonymous referees for several suggestions that vastly improved the paper. For helpful comments, we also thank Jeff Ely, Akifumi Ishihara, Michihiro Kandori, Botond Kőszegi, Simon Loertscher, Marco Pagnozzi, Daisuke Nakajima and audiences at the 2019 Sydney Workshop on Experimental Economics and Theory, the 2019 Organizational Economics Summer Symposium in Sydney, University of Tokyo, Queensland University of Technology, University of Technology Sydney, University of Queensland, University of Sydney, the Berlin Theory Colloquium, University of Amsterdam, the 2020 Australasian Economic Theory Workshop in Adelaide, the 2020 EEA virtual congress and the 2021 virtual Hitotsubashi Summer Institute. Rosato gratefully acknowledges financial support from the Australian Research Council (ARC) through the ARC Discovery Early Career Researcher Award DE180100190. von Wangenheim gratefully acknowledges funding by the German Research Foundation (DFG) through CRC TR 224 (Project B03). The first two authors also thank the 'healthy' Café in Manly for its great hospitality while some of the work on this paper was completed.

[†]UTS Business School.

[‡]University of Queensland, Università di Napoli Federico II and CSEF.

[§]University of Bonn.

1 Introduction

The static first-price auction (FPA) and its dynamic counterpart, the Dutch (or descending-price) auction, are among the most prominent auction formats. In the FPA, bidders submit bids in sealed envelopes; the auctioneer collects all bids, and then awards the prize to the highest bidder who pays for it a price equal to her bid. By contrast, in the Dutch auction the auctioneer begins by calling out a price so high that, presumably, no bidder is willing to buy at that price; the auctioneer then gradually lowers this public price until some bidder indicates her willingness to buy the prize at the given price. A central result in auction theory is that these two formats are strategically equivalent. The crucial insight, due originally to Vickrey (1961), is that the information bidders obtain during the Dutch auction does not affect their optimal strategies; therefore, bidders choose their bids solely based on their prior information. Indeed, the equivalence between these two formats holds in many different environments (e.g., with independent or correlated private values, pure common values, interdependent values, affiliated types, etc...), even under risk aversion. The strategic equivalence further implies that the two auction formats generate the same expected revenue. However, evidence from both laboratory and field experiments shows that revenue equivalence may fail. For instance, Lucking-Reiley (1999) conducts a field experiment by selling *Magic* game cards via Internet auctions and reports that the Dutch auction produces 30-percent higher revenues than the FPA. Katok and Kwasnica (2008) obtain similar results in a laboratory experiment when the price in the Dutch auction drops slowly. These studies suggest that the Dutch auction tends to generate more revenue than the FPA, especially if the price clock of the Dutch auction is relatively slow.[1]

In this paper, we provide a novel explanation for the strategic (and hence, revenue) non-equivalence between the FPA and the Dutch auction based on reference-dependent preferences and loss aversion. We analyze both auction formats in a symmetric environment where bidders have independent private values (IPV) and are expectations-based loss averse à la Kőszegi and Rabin (2006, 2007, 2009). We show that loss-averse bidders bid more aggressively in the Dutch auction than in the FPA. Intuitively, the larger the probability with which a loss-averse bidder expects to win the auction, the stronger her incentives to bid high in order to avoid experiencing disappointment from losing the auction. This is what Kőszegi and Rabin (2006) call the "attachment effect". We argue that, although the two auction formats select the same winner, they create different levels of attachment for the bidders. Consider, for instance, a bidder with a fairly low value. When submitting her bid in the FPA, she knows it is quite likely that one of her opponents has a higher value. Thus, she is rather pessimistic about her chances of winning the auction and not very attached to the prize; therefore, she does not have a strong incentive to bid high. In contrast, consider the same bidder participating in a Dutch auction and imagine the clock is only

[1] However, earlier experiments by Coppinger et *al.* (1980) and Cox *et al.* (1982) report higher revenues for FPA than for Dutch; Cox *et al.* (1983) attribute this finding to probability miscalculations in the Dutch auction.

slightly above the price at which she had originally planned to buy. By now she has updated her beliefs about her strongest opponent's value and is very optimistic that it is below hers – after all, if (one of) her opponents had a much larger valuation than hers, they would have already stopped the clock. Thus, she is very much attached to the prize. In this case, the bidder has a strong incentive to raise her bid and stop the clock at an earlier price in order to reduce the chances of experiencing a loss if another bidder stops the clock before her. In other words, the bidding strategy of a loss-averse bidder in the FPA is shaped by the attachment effect arising from her *initial* beliefs about how likely she is to win the auction. In the Dutch auction, in contrast, she becomes increasingly more optimistic about her chances of winning as the auction unfolds; this creates a stronger attachment effect inducing her to bid more aggressively than in the FPA.

As the theoretical equivalence between the Dutch auction and the FPA holds for many different environments, some authors have suggested that its empirical breakdown might be caused by non-standard risk preferences. Karni (1988) is the first to point out that these two formats are equivalent if and only if bidders are expected-utility maximizers. Nakajima (2011) considers bidders whose preferences exhibit the Allais paradox (Allais, 1953) and shows that the Dutch auction systematically yields more revenue than the FPA.[2] Auster and Kellner (2022) obtain the same result for the case of ambiguity-averse bidders. Another strand of literature, however, attributes the breakdown of the FPA-Dutch equivalence to bidders' time preferences. In fact, in those studies where the Dutch auction generates more revenue than the FPA, typically the clock of the Dutch auction moves rather slowly. Katok and Kwasnica (2008) and Carare and Rothkopf (2005) explain this observation by appealing to bidders' impatience. Our model, while featuring non-standard risk preferences, is also related to this second explanation as a slower clock allows more time for bidders' reference points to adjust.[3]

Section 2 describes the auction environment, bidders' preferences, and solution concept. We consider a standard symmetric environment with independent private values and bidders who have expectations-based reference-dependent preferences as in Kőszegi and Rabin (2009). Hence, in addition to classical material utility, a bidder experiences "gain-loss utility" from comparing her material outcomes to a reference point equal to her (rational) beliefs about these outcomes, as well as "news utility" from updating her reference point from old to new beliefs; both gain-loss and news utility attach a higher weight to losses than to equal-size gains. We focus on symmetric equilibria in increasing strategies; thus, the bidder with the highest value wins the auction.

In Section 3 we begin our analysis by characterizing the equilibrium strategy of loss-averse bidders in the FPA. We show that the attachment effect generates an upward pressure on the equilibrium bids. Indeed, because bidders hold their reference point fixed when submitting their bid, they are willing to pay more in order to reduce the chances of losing the auction.

Next, we turn to the Dutch auction. Here, the main intricacy in characterizing the equilibrium

[2] Weber (1982) shows that the opposite holds if bidders' preferences exhibit the *counter* Allais paradox.

[3] For evidence on the adjustment of reference points, see Imas (2016), Heffetz (2021) and Thakral and Tô (2021).

is a form of belief-based time inconsistency that arises even though bidders' preferences are time consistent.[4] That is, the price at which a bidder stops the clock in equilibrium need not be – and in general it is not – the price at which the bidder would have preferred to stop the clock from the outset. This happens because, as the auction unfolds and the reference point adjusts, the bidder is tempted to "surprise" herself by stopping the clock earlier or later than originally planned. In equilibrium, however, the bidder's plan must be consistent with her expectations so that she stops the clock exactly at the price at which she had planned to do so. We show that there can be multiple consistent bidding plans and identify the symmetric plan that provides bidders with the highest utility from an ex-ante perspective.

We then compare the equilibrium strategies of the two formats and show that loss-averse bidders bid more aggressively in the Dutch auction than in the FPA. An immediate corollary is that the Dutch auction raises more revenue than the FPA. Moreover, by combining ours and previous results, we show that the expected revenue of the four standard auction formats ranks as follows: Dutch > FPA = second-price auction (SPA) > English.[5] Indeed, bidders' beliefs about their likelihood of winning at the time of bidding, and hence their attachment, coincide in the static FPA and SPA. By contrast, the attachment effect is the weakest in the English auction where, as the auction progresses, a bidder becomes more pessimistic about her likelihood of eventually winning.

In Section 4 we show the robustness of our main result by analyzing three different variations of our baseline model. First, we consider the case where, in the Dutch auction, a bidder's reference point does not immediately adjust to her current beliefs, but instead follows a more sluggish adjustment process. In particular, we posit that at any point during the auction, the reference point equals a convex combination between the bidder's initial beliefs (i.e., at the beginning of the auction) and her current ones. Next, we consider the opposite scenario where, in the FPA, a bidder immediately adjusts her reference point after submitting a bid; that is, before learning the auction's outcome. For both cases, we find that, again, the Dutch auction raises more revenue than the FPA. Finally, we show that this revenue ranking continues to hold under the solution concept of choice-acclimating personal equilibrium (CPE), where a bidder's reference point immediately adjusts to her action, on and off the equilibrium path (see Kőszegi and Rabin, 2007).

[4] Kőszegi and Rabin (2009) and Pagel (2016, 2017) explore the implications of this belief-based time inconsistency for intertemporal consumption and saving decisions.

[5] Balzer and Rosato (2021) establish the revenue equivalence between the FPA and the SPA, while von Wangenheim (2021) establishes the ranking between the English auction and the SPA. Using a different solution concept, Lange and Ratan (2010) show that the FPA raises more revenue than the SPA, while Eisenhuth (2019) shows that the all-pay auction yields the most revenue among all sealed-bid formats. Using different dynamic models of reference-dependent preferences than ours, Erhart and Ott (2017) compare Dutch and English auctions while Rosato (2022) analyzes sequential sealed-bid auctions. Fugger *et al.* (2020) characterize the optimal two-stage procurement auction. For related applications of reference-dependent preferences in industrial organization, contract theory, and matching see Heidhues and Kőszegi (2008, 2014), Herweg *et al.* (2010, 2018), Herweg and Mierendorff (2013), Karle and Peitz (2014, 2017), Daido and Murooka (2016), Rosato (2016, 2017), Karle and Schumacher (2017), Macera (2018), Dreyfuss *et al.* (2021), and Meisner and von Wangenheim (2020).

Section 5 concludes the paper by discussing some implications of our results. In particular, we highlight that with expectations-based reference-dependent preferences, the dynamic evolution of beliefs endogenously impacts a bidder's valuation. Hence, the "Revelation Principle" (Myerson, 1979; 1981) does not necessarily hold since, when moving from a direct static mechanism to a dynamic one, a bidder's valuation may change. Therefore, a revenue-maximizing seller should opt for dynamic mechanisms which induce bidders to have optimistic beliefs, thereby increasing their willingness to pay. Finally, we discuss other contexts, beyond auctions, where the attachment effect is likely to play a role, like the dissolution of partnerships, bargaining and dynamic pricing.

2 The Model

In this section, we describe the environment, bidders' preferences, and solution concept.

2.1 Environment

An indivisible item is auctioned off to $N \geq 2$ bidders. Each bidder $i \in \{1, \ldots, N\}$ has a private value $\theta_i \in \Theta := \left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}_+$. Values (or types) are independently and identically distributed across bidders according to a CDF $F : \Theta \to [0,1]$ admitting a continuous PDF f. Let F_1 and f_1 respectively denote the CDF and PDF of the highest order statistic among $N-1$; similarly, let $F_1(\cdot|x)$ and $f_1(\cdot|x)$ respectively denote its CDF and PDF conditional on being lower than x.

In the FPA, bidders simultaneously submit sealed bids; the highest bidder wins the auction and pays her bid. Regarding the Dutch auction, we assume that the clock starts at some sufficiently high price and then drops in steps of size $\varepsilon > 0$. The first bidder who stops the clock wins the auction and pays the price displayed on the clock.

2.2 Preferences and Solution Concept

Consider bidder i bidding b_i in either the FPA or the Dutch auction. If she wins the auction, she obtains an item she values θ_i and pays the price b_i; denote this outcome by (θ_i, b_i). If she loses the auction, she gets nothing and pays nothing; denote this outcome by $(0, 0)$. Hence, the set of material outcomes is $\widetilde{\mathcal{O}} = \{(\theta_i, b_i), (0, 0)\}$ and the bidder's possible material payoffs are $\theta_i - b_i$ and 0, respectively. Following Kőszegi and Rabin (2006, 2007, 2009) we assume that, in addition to classical material utility, the bidder also derives psychological gain-loss utility from comparing her material outcomes to a reference outcome given by her recent expectations (probabilistic beliefs).[6]

[6] For experimental evidence on Kőszegi and Rabin's model see Abeler *et al.* (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Banerji and Gupta (2014), Heffetz and List (2014), Karle *et al.* (2015), Sprenger (2015), Zimmermann (2015), Gneezy *et al.* (2017), Smith (2019), Cerulli-Harms *et al.* (2019), and Rosato and Tymula (2019). For evidence from the field, see Pope and Schweitzer (2011), Card and Dahl (2011) and Crawford and Meng (2011). While most of the evidence indicates that expectations play an important role in shaping reference points, a few studies have also documented some violations of the model's directional predictions.

We start by elaborating on the reference point of the bidder *at the beginning* of the auction. Suppose all bidders $j \neq i$ follow the symmetric, increasing, and differentiable strategy $\beta : \Theta \to \mathbb{R}_+$. If bidder i plans to bid b_i, her reference outcomes are $\mathcal{O} = \{(\theta_i, b_i), (0,0)\}$ and her reference point is a distribution over the set of reference outcomes $\mathcal{O}$. Since β is increasing, at the beginning of the auction bidder i expects to win with probability

$$\Pr\left[\max_{j\neq i} \beta\left(\theta_j\right) < b_i\right] = \Pr\left[\beta\left(\max_{j\neq i} \theta_j\right) < b_i\right] = \Pr\left[\max_{j\neq i} \theta_j < \beta^{-1}\left(b_i\right)\right] = F_1\left(\beta^{-1}\left(b_i\right)\right);$$

hence, her initial reference point is given by

$$r_i = \begin{cases} (\theta_i, b_i) & \text{with probability} \quad F_1\left(\beta^{-1}\left(b_i\right)\right) \\ (0,0) & \text{with probability} \quad 1 - F_1\left(\beta^{-1}\left(b_i\right)\right) \end{cases}.$$

Moreover, the bidder updates her reference point based on the arrival of new information about her material outcomes. In the static FPA, updating only takes place once the auction is over and the bidder learns whether or not she won so that her beliefs become degenerate. In the Dutch auction, instead, at each price drop the bidder observes whether an opponent stopped the clock and instantaneously updates her beliefs about the opponents' types (and hence her likelihood of winning) accordingly.[7] More precisely, suppose that at some price $b' \in \left[\beta\left(\underline{\theta}\right), \beta\left(\overline{\theta}\right)\right]$ the auction is still running. Then, since β is continuously increasing, there is exactly one type $\theta' \in \Theta$ for which $\beta(\theta') = b'$. In turn, bidder i updates her likelihood of winning — given her plan to stop the clock at price b_i — to $F_1(\beta^{-1}\left(b_i\right)|\theta')$. Similarly, a bidder updates her reference point if she decides to deviate to another strategy. For instance, if at price $b' > b_i$ bidder i decides to deviate from the plan to stop the clock at price b_i to the plan of stopping the clock at $\widehat{b}_i$, then she instantaneously updates her likelihood of winning (and thus her reference point) to $F_1(\beta^{-1}\left(\widehat{b}_i\right)|\theta')$.

Such updating of the reference point by itself induces psychological gains and/or losses. In particular, following Kőszegi and Rabin (2009), we assume that the bidder makes an "ordered comparison" percentile-by-percentile between her previous beliefs and her new ones.[8] Formally, for any $p \in (0,1)$ let $c_r(p)$ and $c_{\tilde{r}}(p)$ denote the consumption levels at percentile p under two reference point's distributions r and $\tilde{r}$, respectively. The gain-loss utility arising from updating the reference point from $\tilde{r}$ to r in dimension $k \in \{g, m\}$ is defined as follows:

$$N(r, \tilde{r}) = \sum_{k\in\{g,m\}} \int_0^1 \mu^k(c_r(p) - c_{\tilde{r}}(p))dp.$$

[7] Instantaneous updating implies that the bidder's reference point coincides with her *most* recent, i.e. current, beliefs. In Section 4 we consider a more sluggish adjustment process and show that the main insights are unchanged.

[8] Kőszegi and Rabin (2009) call this "news utility" or "prospective gain-loss utility" and allow this gain-loss utility to be discounted depending on how far in the future consumption outcomes will materialize. Since in our model the outcome will materialize soon, when the auction ends, we abstract away from this possibility and assume that bidders place the same weight on prospective and contemporaneous gain-loss utility. For a different definition of prospective gain-loss utility see Pagel (2019).

Following most of the literature, we assume that the gain-loss function μ^k is piecewise linear:

$$\mu^k(x) = \begin{cases} \eta^k x & \text{if} \quad x \geq 0 \\ \eta^k \lambda^k x & \text{if} \quad x < 0 \end{cases}$$

with $\eta^k > 0$ and $\lambda^k > 1$ for $k \in \{g, m\}$.[9]

We can then obtain an expression for the gain-loss utility of a type-θ bidder from a price drop in the Dutch auction when bidders follow the symmetric strategy β. As long as the price is above $\beta(\overline{\theta})$, any price drop is uninformative and induces no gain-loss utility. Suppose instead that the clock of the Dutch auction drops from price $\beta(\theta')$ to price $\beta(\theta'')$ for $\theta', \theta'' \in [\underline{\theta}, \overline{\theta}]$. If no opponent buys in this time interval, then the probability with which the bidder expects to win increases by $F_1(\theta|\theta'') - F_1(\theta|\theta')$. Hence, the bidder experiences a gain in the item dimension and a loss in the money dimension equal to:

$$N = \eta^g \left[F_1(\theta|\theta'') - F_1(\theta|\theta')\right] \theta - \eta^m \lambda^m \left[F_1(\theta|\theta'') - F_1(\theta|\theta')\right] \beta(\theta).$$

Let $U_{DA}(\tilde{b}|\theta, b')$ denote a type-θ bidder's total expected utility in the Dutch auction when the current clock price is b' and the bidder — who had planned to stop the clock at price $\beta(\theta) < b'$ — is considering to deviate by stopping the clock at price $\tilde{b} < b'$. Apart from material utility, $U_{DA}(\tilde{b}|\theta, b')$ consists of psychological gain-loss utility from both (i) the update of the winning probability due to the deviation to $\tilde{b}$, and (ii) the bidder's expected gain-loss at all future price drops. Similarly, let $U_{FPA}(\tilde{b}|\theta)$ denote a type-θ bidder's total expected utility in the FPA when, having planned to bid $\beta(\theta)$, the bidder is considering deviating by submitting a bid equal to $\tilde{b}$. As a bidder's reference point depends (also) on her own strategy, following Kőszegi and Rabin (2006, 2007, 2009) we require individual strategies to be internally consistent:

Definition 1. *In the FPA, a strategy $\beta(\theta)$ is a personal equilibrium (PE) for a bidder with type θ if, taking as given the distribution of the reference point induced by $\beta(\theta)$, it holds that*

$$U_{FPA}(\beta(\theta)|\theta) \geq U_{FPA}(\tilde{b}|\theta),$$

for any $\tilde{b} \neq \beta(\theta)$.

In the Dutch auction, a strategy $\beta(\theta)$ is a personal equilibrium (PE) for a bidder with type θ if, taking as given the distribution of the reference point induced by $\beta(\theta)$, for all $b' \geq \beta(\theta)$ it holds that

$$U_{DA}(\beta(\theta)|\theta, b') \geq U_{DA}(\tilde{b}|\theta, b'),$$

for any credible deviation $\tilde{b} < b'$.

[9] Although, most of the literature assumes that $\eta^g = \eta^m$ and $\lambda^g = \lambda^m$, we do not impose such a "universal" gain-loss function. Indeed, in Section 3 we will often provide the intuition behind our results by assuming $\eta^m = 0$.

Note the difference between the FPA and the Dutch auction in the conditions for a bidding strategy to be a personal equilibrium. In the FPA, we impose no restrictions on the deviation $\tilde{b}$; this is because bidders submit sealed bids at the beginning of the auction, and hence have only one opportunity to deviate. Indeed, for the FPA we posit that the updating of the reference point takes place only after the auction is over. This assumption reflects the static nature of the FPA. Furthermore, it makes our equilibrium concept for the FPA coincide with the frequently used concept of an "unacclimating" personal equilibrium (UPE) introduced by Kőszegi and Rabin (2007).[10] For the Dutch auction, we also require the deviation $\tilde{b}$ to be *credible*. In particular, we say that a bidder's deviation to stop the clock at price $\tilde{b}$ is credible if she buys when the clock reaches price $\tilde{b}$, given her updated reference point at time $\tilde{b}$. The restriction to credible deviations is important since in the Dutch auction a bidder has multiple opportunities to deviate from her original plan. Indeed, notice that at price b' a bidder might be tempted to deviate from her equilibrium strategy of stopping the clock at price $\beta(\theta)$ to an alternative strategy — such as, for instance, stopping the clock at some other price $\widehat{b}$ — even though she would not carry through with this plan when it is time to execute it. The reason is that the bidder might enjoy additional psychological gain-loss utility from the change in the reference point caused by non-credible deviations; once the reference point has adjusted to the new plan, however, the bidder might want to deviate again (and again...). Hence, the restriction to credible deviations ensures that a bidder will only entertain a plan that she is willing to follow through given the reference point implied by the plan.

Moreover, note that by our restriction to equilibria where bidders use a symmetric and monotone strategy β, we can without loss of generality restrict attention to deviations of the form $\tilde{b} = \beta(\tilde{\theta})$ with $\tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$. Indeed, any bid larger than $\beta(\overline{\theta})$ is dominated by the deviation to $\beta(\overline{\theta})$ as this deviation also leads to winning the auction with certainty, but at a lower price. Similarly, any bid lower than $\beta(\underline{\theta})$ would result in a winning probability of zero, thereby yielding the same payoff as a bid equal to $\beta(\underline{\theta})$. Therefore, by the intermediate value theorem, for any $\tilde{b} \in [\beta(\underline{\theta}), \beta(\overline{\theta})]$ there is a unique $\tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that $\beta(\tilde{\theta}) = \tilde{b}$. Hence, hereafter it will be convenient to think about deviations as mimicking other types, and we will denote with $U_{DA}(\tilde{\theta}|\theta, \theta')$ the expected utility in the Dutch auction of a type-θ bidder who at price $\beta(\theta')$ deviates to the bid $\beta(\tilde{\theta})$; similarly, $U_{FPA}(\tilde{\theta}|\theta)$ will denote the expected utility in the FPA of a type-θ bidder who deviates to the bid $\beta(\tilde{\theta})$. We can now define our solution concept for both auction games:

Definition 2. *A bidding function β constitutes a symmetric personal equilibrium if for each type θ, given the knowledge that opponents bid according to β, the strategy $\beta(\theta)$ is a personal equilibrium.*[11]

[10] An alternative modelling choice would be to separate the updates from the deviation and the resolution of uncertainty, as in the dynamic Dutch auction. In Section 4.2, we show that our main results continue to hold under this alternative modelling choice.

[11] For the FPA, our solution concept is a Bayesian version of the "personal Nash equilibrium" defined by Dato *et al.* (2017), with the additional restriction to symmetric strategies.

Finally, as there can be multiple symmetric personal equilibria, we assume bidders collectively select the one yielding the highest utility from an ex-ante perspective — the "preferred personal equilibrium" (PPE).[12]

3 Analysis

In this section, we derive the equilibrium bidding strategies in the FPA and Dutch auction, and highlight how the attachment effect shapes the incentives of loss-averse bidders. In particular, the magnitude of the attachment effect depends on how optimistic a bidder is at the time of submitting her bid; this, in turn, will imply that the Dutch auction raises more revenue than the FPA.

3.1 Equilibrium Bidding in the FPA

Consider a type-θ bidder who has planned to bid $\beta_{FPA}(\theta)$ but deviates by mimicking a bidder with type $\tilde{\theta} \geq \theta$.[13] In this case, her expected payoff is:

$$\begin{aligned} U_{FPA}(\tilde{\theta}|\theta) \;=\; & F_1(\tilde{\theta})\left[\theta - \beta_{FPA}(\tilde{\theta})\right] - \eta^g\lambda^g\left[1 - F_1(\tilde{\theta})\right]F_1(\theta)\theta + \eta^g F_1(\tilde{\theta})\left[1 - F_1(\theta)\right]\theta \\ & + \eta^m\left[1 - F_1(\tilde{\theta})\right]F_1(\theta)\beta_{FPA}(\theta) - \eta^m\lambda^m F_1(\tilde{\theta})\left[1 - F_1(\theta)\right]\beta_{FPA}(\tilde{\theta}) - \eta^m\lambda^m F_1(\tilde{\theta})F_1(\theta)\left[\beta_{FPA}(\tilde{\theta}) - \beta_{FPA}(\theta)\right]. \end{aligned} \tag{1}$$

The first term in (1) represents the standard expected material payoff. The other terms capture expected gain-loss utility and are derived as follows. The second term captures the loss in the item dimension for a bidder who expected to win the auction with probability $F_1(\theta)$ but ends up losing it — an event happening with probability $1-F_1(\tilde{\theta})$ — and thus experiences a loss equal to $\eta^g\lambda^g F_1(\theta)\theta$. Similarly, the third term captures the gain in the item dimension for a bidder who expected to lose with probability $1 - F_1(\theta)$ but ends up winning — an event happening with probability $F_1(\tilde{\theta})$ — and thus experiences a gain equal to $\eta^g\left[1 - F_1(\theta)\right]\theta$. The fourth and fifth terms capture the corresponding expected gains and losses in the money dimension. The final term captures the loss in the money dimension when winning at a price higher than expected. Differentiating (1) with respect to $\tilde{\theta}$ and evaluating the resulting first-order condition at $\tilde{\theta} = \theta$ yields a differential equation whose solution provides us with the equilibrium bidding strategy:[14]

Proposition 1. *The symmetric PPE bidding strategy in the FPA is given by*

$$\beta_{FPA}(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1 + \eta^g\lambda^g F_1(x) + \eta^g\left[1 - F_1(x)\right]}{F_1(\theta)\left(1 + \eta^m\lambda^m\right)} e^{\frac{\eta^m(\lambda^m-1)[F_1(\theta)-F_1(x)]}{1+\eta^m\lambda^m}} x f_1(x)dx. \tag{2}$$

[12] Notice that Kőszegi and Rabin (2006, 2007, 2009) propose PPE to address the issue of the multiplicity of personal equilibria in the context of *individual* decision problems. In our multi-player game, however, selecting the PPE is akin to assuming that all bidders are able to coordinate on the (symmetric) personal equilibrium that is best for the group. Notwithstanding this additional restriction, we think the PPE selection represents a reasonable benchmark in our model as it provides the auctioneer with a worst-case scenario.

[13] As shown by Balzer and Rosato (2021), upward deviations are the most relevant ones.

[14] The FOC is also sufficient since the bidder's payoff satisfies single crossing; see Balzer and Rosato (2021).

Balzer and Rosato (2021) derived the symmetric PPE bidding function for an environment with interdependent values and independent signals. As the IPV model is a special case of theirs, applying their result to our environment yields expression (2). It is easy to verify that $\beta_{FPA}(\theta)$ is increasing in the coefficient of loss aversion in the item dimension (λ^g) and decreasing in the coefficient of loss aversion in the money dimension (λ^m). Intuitively, if the bidder wins the auction she experiences a loss in money; this induces her to reduce her bid when loss aversion in the money dimension becomes stronger. Similarly, the bidder experiences a loss in the item dimension when she loses the auction; this, in turn, induces her to increase her bid when loss aversion in the item dimension becomes stronger.

Next, we compare the loss-averse bidding strategy with the risk-neutral benchmark. For $\eta^m = 0$, expression (2) reduces to

$$\beta_{FPA}(\theta) = \mathbb{E}_{F_1}[v(x)|x \leq \theta],$$

where

$$v(x) = \{1 + \eta^g \lambda^g F_1(x) + \eta^g [1 - F_1(x)]\} x.$$

The term $v(x)$ represents the belief-dependent "opportunity value" of winning the auction for a bidder with type x. Indeed, in addition to classical material utility, when winning the bidder also experiences a gain equal to $\eta^g [1 - F_1(x)] x$ while concurrently avoiding a loss equal to $\eta^g \lambda^g F_1(x)x$. Hence, as in the risk-neutral benchmark, bidders in equilibrium bid the expectation of their strongest opponent's valuation conditional on winning; with expectations-based loss aversion, however, this valuation equals the opportunity value of winning which also depends on the bidder's beliefs. Importantly, the belief-dependent part of this opportunity value increases in a bidder's type. This is the attachment effect: bidders with higher types expect to win with a higher probability and thus feel more attached to the prize. As a result, compared to the risk-neutral benchmark, high types overbid more strongly than low types.[15]

Depending on the magnitude of the gain-loss parameters, bidders may even bid more than their intrinsic value θ, so that they attain a negative material payoff in expectation. Yet, such behavior can be optimal since an aggressive bid may prevent an even larger psychological loss from losing the auction. At first, this may seem at odds with the fact that a bidder would receive zero utility by bidding zero and abstaining from the auction altogether. However, bidding zero is not a personal equilibrium for a bidder with a strictly positive value since, fixing her expectations, the bidder would always be tempted to deviate and submit a positive bid in order to enjoy an unexpected psychological gain.[16]

[15] Straight overbidding compared to risk neutrality is driven by the assumption that $\eta^m = 0$, which reduces the weight over money relative to the item dimension in a bidder's utility. Yet, the intuition that a stronger attachment effect leads to more aggressive bids for high types equally applies when bidders are loss averse in both dimensions.

[16] The idea that loss-averse buyers may be hurt by being exposed to offers that they cannot credibly refuse is a common theme in a related literature in behavioral industrial organization. In particular, firms may explicitly exploit this behavior by inducing a strong attachment via, for instance, random sales prices (Heidhues and Kőszegi, 2014), informational advertisement (Karle and Schumacher, 2017), or limited supply of bargains (Rosato, 2016).

3.2 Equilibrium Bidding in the Dutch Auction

Differently from the FPA, the Dutch auction is a dynamic format where a bidder's beliefs (and hence her reference point) evolve throughout the auction. Moreover, when she submits her bid in the FPA, the bidder is unsure about whether she will win; in the Dutch auction, instead, when she submits her bid by stopping the clock, the bidder is sure to win.

In a symmetric equilibrium, a type-θ bidder stops the clock at price $\beta_{DA}(\theta)$. In particular, the bidder prefers executing this plan over switching to another credible plan at any point in time. Suppose the current clock price is $\beta_{DA}(\theta') > \beta_{DA}(\theta)$ and a type-θ bidder considers deviating to another plan, $\beta_{DA}(\tilde{\theta}) > \beta_{DA}(\theta)$.[17] In this case, her expected payoff is:

$$\begin{aligned} U_{DA}(\tilde{\theta}|\theta,\theta') \;&=\; F_1(\tilde{\theta}|\theta')[\theta-\beta_{DA}(\tilde{\theta})]+\eta^g\theta\left[F_1(\tilde{\theta}|\theta')-F_1(\theta|\theta')\right] \\ &\quad -\eta^m\lambda^m[F_1(\tilde{\theta}|\theta')\beta_{DA}(\tilde{\theta})-F_1(\theta|\theta')\beta_{DA}(\theta)]+\mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right], \end{aligned} \tag{3}$$

where $\mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right]$ is the sum of total expected news utility a type-θ bidder expects to experience from all belief updates between price $\beta_{DA}(\theta')$ and price $\beta_{DA}(\tilde{\theta})$ given the new plan to buy at price $\beta_{DA}(\tilde{\theta})$ (its explicit expression is derived in Lemma 1 below). The first term on the right-hand side of (3) is the standard expected material payoff. The other terms capture expected gain-loss utility. By deviating from her plan to buy at price $\beta_{DA}(\theta)$ to the new plan of buying at price $\beta_{DA}(\tilde{\theta})$, the bidder's probability of winning increases from $F_1(\theta|\theta')$ to $F_1(\tilde{\theta}|\theta')$. Hence, by deviating she experiences a gain in the item dimension equal to $\eta^g\theta[F_1(\tilde{\theta}|\theta')-F_1(\theta|\theta')]$. At the same time, however, the bidder also increases her expected payment from $F_1(\theta|\theta')\beta_{DA}(\theta)$ to $F_1(\tilde{\theta}|\theta')\beta_{DA}(\tilde{\theta})$, thereby experiencing a loss in the money dimension equal to $\eta^m\lambda^m[F_1(\tilde{\theta}|\theta')\beta_{DA}(\tilde{\theta})-F_1(\theta|\theta')\beta_{DA}(\theta)]$. The last term on the right-hand side of (3) captures news utility; that is, the expected gain-loss utility stemming from changes in beliefs and the resulting updating of the reference point as the auction unfolds. The next result allows us to re-write this expression in terms of the model's primitives.

Lemma 1. *Let the current clock price be $\beta_{DA}(\theta')$ and consider a bidder of type θ planning to stop the clock at price $\beta_{DA}(\tilde{\theta}) < \beta_{DA}(\theta')$. For $\varepsilon \to 0$, the following equality holds:*

$$\mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right] = -\left[\eta^g(\lambda^g-1)\theta+\eta^m(\lambda^m-1)\beta_{DA}(\tilde{\theta})\right]\int_{\tilde{\theta}}^{\theta'} f_1(x|\theta')F_1(\tilde{\theta}|x)dx. \tag{4}$$

In order to keep the analysis tractable and simplify the notation as much as possible, for the remainder of the analysis we will focus on the limit case where $\varepsilon \to 0$, which can be interpreted as an arbitrarily fine price grid. The term on the right-hand side of (4) is a natural generalization of static expected gain-loss utility to a dynamic setting. We discuss it by focusing on the risk in the item dimension, but a similar intuition applies for the money dimension. From the perspective of a bidder who is active at price $\beta_{DA}(\theta')$, at any future price $\beta_{DA}(x) \in \left[\beta_{DA}(\tilde{\theta}), \beta_{DA}(\theta')\right]$ only

[17] As for the FPA, upward deviations are the most relevant ones.

one of the two following events can realize. The auction may continue and the bidder learns that her strongest opponent's type is below x. This event, given the current price is $\beta_{DA}(\theta')$, happens with probability $F_1(x|\theta')$; in this case, the bidder updates her beliefs and her probability of winning increases by $-\frac{\partial}{\partial x}F_1(\tilde{\theta}|x) = f_1(x|x)F_1(\tilde{\theta}|x)$, generating a gain equal to $\eta^g f_1(x|\theta')F_1(\tilde{\theta}|x)$. Alternatively, the auction may end and the bidder learns that her strongest opponent's type is exactly x. This event, given the current price is $\beta_{DA}(\theta')$, happens with (marginal) probability $f_1(x|\theta')$; in this case, she learns that she lost and her beliefs about winning drop from $F_1(\tilde{\theta}|x)$ to zero, generating a loss equal to $\eta^g\lambda^g f_1(x|\theta')F_1(\tilde{\theta}|x)$.

An equilibrium bid is a credible plan about when to stop the clock such that, at any point during the auction, the bidder prefers executing it over switching to another credible plan. Verifying that an equilibrium bid is indeed a credible plan is technically tedious and so we relegate it to Appendix A. Yet, equilibrium behavior is rather intuitive: at any price $\beta_{DA}(\theta') > \beta_{DA}(\theta)$ a type-θ bidder prefers to stay in the auction instead of buying immediately; hence, $U_{DA}(\theta|\theta,\theta') \geq U_{DA}(\theta'|\theta,\theta')$. In Appendix A we show that letting $\theta' \to \theta$ yields a lower bound on the derivative of the bidding function. Letting this lower bound bind and solving the resulting differential equation provides us with the equilibrium bidding strategy:[18]

Proposition 2. *The symmetric PPE bidding strategy in the Dutch auction is given by*

$$\beta_{DA}(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1+\eta^g\lambda^g}{F_1(\theta)\left(1+\eta^m\lambda^m\right)} \left[\frac{F_1(\theta)}{F_1(x)}\right]^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}} x f_1(x)dx. \tag{5}$$

Again, it is easy to see that $\beta_{DA}(\theta)$ is increasing in the coefficient of loss aversion in the item dimension (λ^g) and decreasing in the coefficient of loss aversion in the money dimension (λ^m). Moreover, for $\eta^m = 0$ expression (5) simplifies to

$$\beta_{DA}(\theta) = (1+\eta^g\lambda^g)\mathbb{E}_{F_1}[x|x \leq \theta].$$

Hence, compared to the risk-neutral benchmark, every type overbids by a factor of $1+\eta^g\lambda^g$. As in the FPA, bidders bid the expectation of their strongest opponent's opportunity value of winning, where this value is now given by

$$v(x) = (1+\eta^g\lambda^g)x.$$

Indeed, bidding behavior in the Dutch auction is driven by a bidder's incentives shortly before she buys and, at this time, the bidder is effectively certain to win. This magnifies the attachment effect (and therefore the opportunity value of winning) compared to the FPA, where a bidder's

[18] The solution to the differential equation represents only a necessary condition for equilibrium; in the proof of Proposition of Proposition 2 in Appendix A, we show that this solution also satisfies the sufficient conditions for PE, and that the PPE is given by the PE that entails the lowest bid.

attachment is pinned down by her ex-ante beliefs.

The fact that bidding strategy in the Dutch auction is different than in the FPA immediately suggests that bidders' ex-ante utility will also differ between the two formats. Remarkably, however, loss-averse bidders would be worse off in the Dutch auction *even if they were to bid the same in both formats*. The next proposition formally states this result.

Proposition 3. *If bidders use the same strategy in the FPA and the Dutch auction, their ex-ante utility is lower in the Dutch auction than in the FPA.*

The intuition for the above result is as follows. If bids are the same in both formats, a bidder's expected payment and probability of winning are also the same; this, in turn, implies that from an ex-ante perspective a bidder's reference points coincide in the two formats. Yet, while in the FPA uncertainty is resolved all at once, the Dutch auction entails a more gradual resolution of uncertainty. Such a gradual resolution of uncertainty exposes a bidder to the risk of first becoming more optimistic and then suddenly learn that an opponent just bought the item. A loss-averse bidder dislikes these fluctuations in beliefs. Indeed, if possible, the bidder would prefer to commit to a bid at the beginning and avoid seeing the auction unfold.[19]

The dislike for fluctuations in beliefs is also the source of a belief-based form of time inconsistency that arises in the bidders' plans even though their preferences are time consistent. In particular, bidders with rather low types, who are exposed to fluctuations in beliefs for a relatively long time, would like to mitigate the expected losses from such fluctuations during the auction by committing to a lower bid. Indeed, as the next proposition shows, such deviation would make low-type bidders better off.

Proposition 4. *In the Dutch auction there exists a cut-off type $\theta^* \in (\underline{\theta}, \overline{\theta})$ such that all types $\theta < \theta^*$ would ex-ante increase their utility by deviating to a lower bidding strategy.*

Yet, such a mitigation strategy is not dynamically consistent and, therefore, cannot be part of an equilibrium. The reason is that, when a bidder decides to stop the clock, her losses from the gradual resolution of uncertainty are "sunk" and hence do not affect her incentives. Hence, even though a bidder may ex-ante prefer a lower bidding strategy, she anticipates that she would never actually execute such a plan.

The results in both Proposition 3 and Proposition 4 reveal a demand for commitment in the Dutch auction on the part of loss-averse bidders. Such a commitment could be achieved via proxy bidding whereby bidders could effectively transform the Dutch auction into a first-price one. Yet, while auction sites like eBay and others usually provide proxy bidding services in English (or ascending-price) auctions — effectively turning them into second-price ones — such services are much less common for Dutch auctions. Indeed, in the next section we will see that it is in the

[19] Pagel (2018) makes a similar observation for a loss-averse investor who prefers to ignore and not rebalance his portfolio because he dislikes bad news more than he likes good news.

seller's interest for bidders to engage in proxy bidding in English auctions, but not Dutch ones; hence, our model provides a potential reason why proxy bidding is much more prevalent in English than in Dutch auctions.[20]

3.3 Revenue Comparison

We now show that, by creating a stronger attachment effect, the Dutch auction raises more revenue than the FPA. Suppose $\eta^m = 0$ and consider a type-θ bidder who contemplates mimicking type $\theta' > \theta$. In equilibrium, $U_{FPA}(\theta|\theta) \geq U_{FPA}(\theta'|\theta)$; hence, using expression (1), the following must hold:

$$F_1(\theta')\beta_{FPA}(\theta') - F_1(\theta)\beta_{FPA}(\theta) \geq [F_1(\theta') - F_1(\theta)]\,\theta + \eta^g\theta[F_1(\theta') - F_1(\theta)]\,[1 + (\lambda^g - 1)\,F_1(\theta)]\,. \quad (6)$$

Similarly, for the Dutch auction, in equilibrium it holds that $U_{DA}(\theta|\theta,\theta') \geq U_{DA}(\theta'|\theta,\theta')$ for any price $\beta_{DA}(\theta') > \beta_{DA}(\theta)$; multiplying both sides in (3) by $F_1(\theta')$ and re-arranging yields:

$$\begin{aligned} F_1(\theta')\beta_{DA}(\theta') - F_1(\theta)\beta_{DA}(\theta) \;\; &\geq \;\; [F_1(\theta') - F_1(\theta)]\,\theta \\ &+\eta^g\theta[F_1(\theta') - F_1(\theta)] + \eta^g(\lambda^g - 1)\theta F_1(\theta')\int_\theta^{\theta'} F_1(\theta|x)f_1(x|\theta')dx. \quad (7)\end{aligned}$$

In both (6) and (7), the term on the left-hand side and the first term on the right-hand side represent the familiar material costs and benefits associated with mimicking a bidder with a higher type — trading off a higher probability of winning against paying a higher price. The additional terms on the right-hand side represent the additional incentives that a loss-averse bidder has to raise her bid; i.e., to realize gains and/or avoid losses. These incentives are stronger in the Dutch auction because

$$F_1(\theta')\int_\theta^{\theta'} F_1(\theta|x)f_1(x|\theta')dx > \int_\theta^{\theta'} F_1(\theta)f_1(x)dx = [F_1(\theta') - F_1(\theta)]F_1(\theta).$$

The incentives to raise one's bid are stronger in the Dutch auction since, when doing so, a bidder is (almost) sure to win; hence, by bidding more aggressively, she can reduce the expected losses caused by the fluctuations in beliefs that arise when updating the reference point.[21] Indeed, we have the following result:

Proposition 5. *$\beta_{DA}(\theta) \geq \beta_{FPA}(\theta)$ and this inequality is strict for all $\theta > 0$.*

[20] Pricefalls is the only auction site running Dutch auctions with proxy bidding — called "buy if it hits" — that we are aware of. The company was founded in 2009 in response to users' frustrations with the eBay model and with a pronounced emphasis on buyer's satisfaction.

[21] This effect similarly applies to gain-loss utility over money since, by stopping the clock earlier, a bidder also avoids fluctuations in her expected payment.

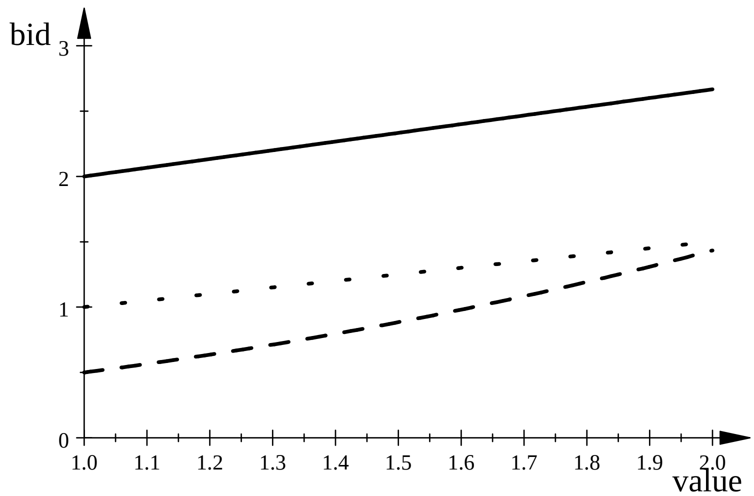

Figure 1: Bidding functions in the FPA (dashed) and Dutch auction (solid) for $N = 2$, $\eta^g = \eta^m = 1$ and $\lambda^g = \lambda^m = 3$ compared to risk neutrality (dotted) with θ distributed uniformly on $[1, 2]$.

Thus, loss-averse bidders bid more in the Dutch auction than in the FPA. The next result then follows immediately from Proposition 5.

Corollary 1. *With loss-averse bidders the Dutch auction yields a higher revenue than FPA.*

The attachment effect in the FPA depends on a bidder's ex-ante likelihood of winning. In the Dutch auction, instead, the attachment effect grows over time. Indeed, as the price at which a bidder had planned to stop the clock approaches, her beliefs about her chances of winning — and hence her willingness to pay — increase. This, in turn, pushes a bidder to (plan to) stop the clock at a price which is higher than her bid in the FPA.

Figure 1 displays the loss-averse bidding strategies in the Dutch auction and the FPA, along with the risk-neutral one, for the case of equal gain-loss utility across dimensions. Notice that $\beta_{FPA}(\underline{\theta}) = \left(\frac{1+\eta}{1+\eta\lambda}\right)\underline{\theta}$ while $\beta_{DA}(\underline{\theta}) = \left(\frac{1+\eta\lambda}{1+\eta}\right)\underline{\theta}$. These bids coincide with the maximum price at which a loss-averse buyer with intrinsic valuation $\underline{\theta}$ is willing to buy when her expectations are to never or, respectively, always get the item (see Kőszegi and Rabin, 2006). Hence, at the time of submitting her bid, the attachment effect of a bidder with the lowest type is at its minimum in the FPA, but at its maximum in the Dutch auction. Moreover, Figure 1 also shows that, with uniformly distributed values, in the Dutch auction all types overbid whereas in the FPA all types underbid compared to the risk-neutral benchmark. The next proposition shows that the first observation holds independently of the distribution of values, while the second one is robust only for low types.

Proposition 6. *Let $\eta^k = \eta > 0$ and $\lambda^k = \lambda > 1$ for $k \in \{g, m\}$. In the Dutch auction all types overbid compared to the risk-neutral benchmark. In the FPA there exists a cutoff $\hat{p} \in ((0.5)^{\frac{1}{N-1}}, 1]$ such that a bidder overbids compared to the risk-neutral benchmark if and only if $F(\theta) > \hat{p}$.*

Hence, while low types underbid in the FPA independently of the distribution of values, some types at the top of the distribution might overbid. Yet, as the number of bidders increases, the share of types who underbid approaches one.

Combining Corollary 1 with results by Balzer and Rosato (2021) and von Wangenheim (2021), we obtain that the Dutch auction raises the most revenue among the four main auction formats:

Corollary 2. *With loss-averse bidders, in terms of expected revenue, the four main auction formats can be ranked as follows:*

$$\textit{Dutch} > \textit{FPA} = \textit{SPA} > \textit{English}.$$

Intuitively, the FPA and SPA are revenue equivalent as, since they are both static formats where a bidder's reference point depends on her ex-ante likelihood of winning, they induce the same level of attachment. The English auction raises the least revenue since a bidder becomes less optimistic about her chances of winning as the auction unfolds; this in turn lowers the bidder's reference point, inducing her to bid less aggressively than in the SPA. In other words, while in a Dutch auction a bidder's initial attachment grows as the auction evolves, in an English auction a bidder becomes less attached to the item as the auction continues. Hence, by creating the strongest attachment for bidders, the Dutch auction raises the most revenue among standard formats. The evidence from both the lab and the field is broadly consistent with this ranking. Indeed, Lucking-Reiley (1999) and Katok and Kwasnica (2008) find that the Dutch auction raises more revenue than the FPA. Moreover, several studies show that with private values the SPA tends to raise more revenue than the English auction; see Kagel *et al.* (1987) and Harstad (2000). Finally, Cheema *et al.* (2012) find that the Dutch auction yields a higher revenue than the English auction, and even more so when the clocks of the two auctions are relatively slow.

We conclude this section with a brief discussion on how one could go about separately identifying our explanation for the superiority of Dutch auctions based on expectations-based loss aversion from alternative hypotheses that have been proposed in the literature. Besides ours, there are three main alternative explanations for why the Dutch auction raises more revenue than the FPA: (i) bidder impatience (Carare and Rothkopf, 2005; Katok and Kwasnica, 2008), (ii) bidder preferences that exhibit the Allais paradox (Nakajima, 2011), and (iii) ambiguity-averse bidders (Auster and Kellner, 2022). First, one can differentiate our explanation — as well as (ii) and (iii) — from bidder impatience by increasing the stakes in the auction; e.g., shifting up distribution of types by a constant amount. In this case, under bidder impatience, the difference in bids between the Dutch auction and the FPA would shrink, while it would not according to our model. Next, our explanation can be differentiated from those of Nakajima (2011) and Auster and Kellner (2022) by examining the behavior of low-type bidders. Indeed, our model predicts overbidding in the Dutch auction for bidders with low types; in contrast, in the models of both Nakajima (2011) and Auster and Kellner (2022) no bidder would ever submit a bid larger than her type. Moreover, notice that while the superiority of the Dutch auction holds also when bidders have Allais-type preferences

(Nakajima, 2011) or ambiguity-averse preferences (Auster and Kellner, 2022), the overall ranking of the four auction formats is different. Indeed, the English and second-price auctions continue to be revenue equivalent with ambiguity-averse bidders as well as with bidders who have Allais-type preferences; see Karni and Safra (1989) and Neilson (1994). Yet, as mentioned before, several studies find that with private values the SPA tends to raise more revenue than the English auction. The reason why loss aversion can rationalize this finding is that with expectations-based reference points, bidding one's intrinsic value is not a dominant strategy anymore in neither the English auction nor the SPA; indeed, as shown by Rosato and Tymula (2019), the number of competitors affects the bidding strategy of a loss-averse bidder in the SPA. More generally, ample evidence, gathered from both the field and the lab, shows that reference dependence and loss aversion can rationalize various phenomena across different domains; e.g., the "endowment effect" in laboratory trade experiments and the "disposition effect" in asset or property markets.

4 Extensions and Robustness

In this section, we investigate the robustness of our main result by analyzing three extensions of our baseline model. In the first one, we relax the assumption of instantaneous updating of the reference point in the Dutch auction and consider a more sluggish adjustment process. In the second one, we consider the opposite case where, in the FPA, a bidder immediately adjusts her reference point after submitting her bid. Finally, we derive the bidding strategies under the alternative solution concept of choice-acclimating personal equilibrium (CPE), whereby a bidder's reference point immediately adjust to her bid — capturing the notion that bidders can commit to a bid before uncertainty fully resolves. For all three extensions, we find that the Dutch auction continues to generate a stronger attachment effect — and hence a higher revenue — than the FPA.

4.1 Adjustment Speed of the Reference Point

In our model, bidders in the Dutch auction update their reference points instantaneously; that is, a bidder's reference point coincides with her most recent beliefs. While convenient for the purpose of illustrating the attachment effect, this assumption might be too extreme as some time might be required for the reference point to adjust.[22] However, none of our qualitative results concerning the comparison between the FPA and the Dutch auction hinge on the updating of the reference point being instantaneous. Indeed, as we will show in this section, as long as changes in beliefs cause an update, however small, of the reference point, the revenue ranking between the two auction formats still holds.

[22] The speed of adjustment of reference points is still a subject of debate in the experimental literature. In a real-effort task, Gill and Prowse (2012) find "essentially instantaneous" adjustment of the reference point. Smith (2019), however, suggests that reference points need some time to fully adjust. Heffetz (2021) suggests that beliefs have to "sink in" to become reference points, and the speed at which they sink in heavily depends on the environment.

Consider an active bidder in the Dutch auction who plans to buy at price $\beta_{DA}(\theta)$. A tractable way of modeling the updating of the reference point is to assume that while the auction unfolds, for any clock price $\beta_{DA}(\theta')$, the bidder's reference point is a convex combination between her most recent beliefs $F_1(\theta|\theta')$ and her beliefs at the beginning of the auction, $F_1(\theta)$; then, when the auction terminates, the beliefs about the final allocation "sink in" and each bidder updates her reference point with respect to the final allocation. Let $\alpha \in [0, 1]$ be the weight on a bidder's most recent beliefs in her reference point as the Dutch auction unfolds. Evidently, $\alpha = 1$ corresponds to the situation analyzed in Section 3, where the Dutch auction raises more revenue than the FPA. By contrast, for $\alpha = 0$ there is no updating of the reference point during the Dutch auction so that a bidder's reference point stays equal to her ex-ante beliefs; it's easy to see then that in this case the two auction formats are revenue equivalent. For $\alpha \in (0, 1)$, we have the following result:

Proposition 7. *For all $\theta > \underline{\theta}$ the symmetric PPE bidding strategy $\beta_{DA}(\theta)$ in the Dutch auction is increasing in α.*

Thus, equilibrium bids in the Dutch auction increase in how strongly the reference point adjusts; this, in turn, implies that for any $\alpha > 0$ a loss-averse bidder behaves more aggressively in the Dutch auction than in the FPA. In a controlled laboratory experiment, Katok and Kwasnica (2008) find that the Dutch auction generates more revenue than the FPA if the clock moves rather slowly. This finding is consistent with our model, as a slow clock provides more time for the reference point to adjust; i.e., a slower clock corresponds to a larger α in our model.[23]

4.2 Updating in the First-Price Auction

In our baseline model, we assumed that when a bidder deviates from her equilibrium bid in the FPA, the reference point only updates once after the auction resolves. This assumption seems appropriate if there is little time between placing a bid and the resolution of the auction; in this case, our equilibrium concept coincides with the "unacclimating personal equilibrium" (UPE) of Kőszegi and Rabin (2007). Yet, depending on the auction environment, it may also be plausible that, in case of a deviation, the reference point may adjust to the new bid — and thus induce gain-loss utility — before the auction resolves. In this case, a bidder may experience gains and losses twice: first when deviating from her equilibrium bid, and then once more when the uncertainty resolves. In particular, this alternative formulation might be appropriate if the time span between submitting a bid and learning the outcome of the auction is rather large. However, as we will show in this section, under this alternative timing, the revenue gap between the Dutch auction and the FPA widens.

[23] However, they also find that with a fast clock the FPA raises more revenue than the Dutch auction whereas in our model, for the limiting case of $\alpha = 0$, the two auction formats are revenue equivalent.

In order to describe upward and downward deviations conveniently in one equation let

$$\Lambda^k_{\tilde{\theta}<\theta} := \begin{cases} \eta^k \left(\lambda^k - 1\right) & \tilde{\theta} < \theta \\ 0 & \tilde{\theta} \geq \theta \end{cases}$$

for $k \in \{m, g\}$. With the two separate updates in case of a deviation, the expected utility of a type-θ bidder who deviates by mimicking a bidder of type $\tilde{\theta} \neq \theta$ reads

$$\begin{aligned} U_{FPA}(\tilde{\theta}|\theta) \;=\; & F_1(\tilde{\theta}) \left[\theta - \beta_{FPA}(\tilde{\theta})\right] + \left[F_1(\tilde{\theta}) - F_1(\theta)\right] (\eta^g + \Lambda^g_{\tilde{\theta}<\theta})\theta \\ & - \left[F_1(\tilde{\theta}) - F_1(\theta)\right] (\eta^m \lambda^m - \Lambda^m_{\tilde{\theta}<\theta})\beta_{FPA}(\theta) - F_1(\tilde{\theta})(\eta^m \lambda^m - \Lambda^m_{\tilde{\theta}<\theta}) \left[\beta_{FPA}(\tilde{\theta}) - \beta_{FPA}(\theta)\right] \\ & + \left[1 - F_1(\tilde{\theta})\right] F_1(\tilde{\theta}) \left[-\eta^g \lambda^g \theta + \eta^m \beta_{FPA}(\tilde{\theta})\right] + \left[1 - F_1(\tilde{\theta})\right] F_1(\tilde{\theta}) \left[\eta^g \theta - \eta^m \lambda^m \beta_{FPA}(\tilde{\theta})\right]. \end{aligned} \tag{8}$$

The first two lines on the right-hand side of (8) describe the classical material utility and the gain-loss utility from the change in winning probability and expected price due to the deviation, respectively. The third line describes expected gain-loss utility from the resolution of the auction. It is insightful to compare expression (8) with expression (1) in the baseline model. Compared to the baseline model, in this new specification the bidder is subject to more belief fluctuations. For instance, if a bidder deviates to a higher bid but ends up losing the auction, her beliefs to obtain the item will first jump up, and then eventually drop down to zero. Since losses are weighted more strongly than gains, these fluctuations create a net negative utility compared to the baseline specification with only one update; in turn, this additional negative utility makes deviations to higher bids less attractive. Hence, under this specification, the preferred personal equilibrium lies strictly below the one of the baseline model.

Proposition 8. *Let $\Lambda^k := \eta^k(\lambda^k - 1) \leq 1$ for $k \in \{m, g\}$. With instantaneous updating in the FPA as described in Equation (8), the expected revenue in the FPA is strictly lower than in the baseline model as described in Equation (1), and — a fortiori — lower than in the Dutch auction.*

The condition $\Lambda^k \leq 1$, first introduced introduced in Herweg *et al.* (2010) who coined for it the label "non-dominance of gain-loss utility", ensures that a loss-averse agent does not select first-order stochastically-dominated options.[24] This condition is sufficient for the existence of a separating equilibrium under the solution concept of "choice-acclimating personal equilibrium" (CPE) — see the next subsection for more details — but is not needed under UPE. Indeed, the expression of a bidder's utility in (8) can be interpreted as a combination of UPE and CPE in that the psychological utility stemming from a deviation follows the logic of UPE, while that stemming from the resolution of uncertainty follows the logic of CPE.

[24] See also Masatlioglu and Raymond (2016).

4.3 Solution Concept

Until now, we have employed the solution concept of "unacclimating personal equilibrium" (UPE) to analyze bidding in both the FPA and the Dutch auction. According to this concept, a bidder's equilibrium action determines her reference point and, when deviating to an off-path action, she experiences psychological (dis-)utility from the changed distribution of final outcomes; in other words, in a UPE a bidder keeps her reference point fixed when considering deviations. Moreover, a bidder also experiences psychological (dis-)utility when her beliefs change because of the arrival of new information (i.e. "news utility"). Hence, this solution concept best applies to auctions in which bidders form their plans sufficiently in advance for their expectations to become a reference point, but are not able to commit to a particular bid until shortly before uncertainty is resolved. The Dutch auction, given its dynamic nature, clearly meets these criteria; yet, we think that our solution concept can also apply to sealed-bid auctions, especially if bidders have been looking ahead to the auction for quite some time but can still wait until the last minute to submit their actual bids. Furthermore, in some sealed-bid formats, like so-called "silent" or "secret-bid" auctions, the bidding phase lasts for a prespecified period of time during which bidders are required to be physically present and can revise their (sealed) bids multiple times.

Nevertheless, most of the prior literature on sealed-bid auctions with loss-averse bidders has employed the alternative solution concept, introduced by Kőszegi and Rabin (2007), of "choice-acclimating personal equilibrium" (CPE); see Lange and Ratan (2010) and Eisenhuth (2019). In a CPE, a bidder's reference point immediately adjusts to her action, both on and off the equilibrium path. Let $U_{FPA}(\tilde{\theta}|\tilde{\theta};\theta)$ denote the expected payoff in the FPA of a type-θ bidder who mimics a type-$\tilde{\theta}$ bidder. Then, a bidding function $\beta_{FA}(\theta)$ is a CPE in the FPA if $U_{FPA}(\theta|\theta;\theta) \geq U_{FPA}(\tilde{\theta}|\tilde{\theta};\theta)$. In other words, a bidder fully internalizes how a deviation affects her reference point and she experiences psychological (dis-)utility only from comparing the final realized outcomes to her most recent beliefs; thus, there is no "news utility" in CPE. In what follows, we show that also under CPE the Dutch auction continues to generate a stronger attachment effect and to raise a higher revenue than the FPA.

The original definition of CPE in Kőszegi and Rabin (2007) applies only to static decision problems. Hence, for the Dutch auction, we rely on the concept of sequential CPE (SCPE) — an extension of CPE to dynamic decision problems introduced by Rosato (2022). For any chosen strategy $\beta_{DA}(\theta)$, at each price $\beta_{DA}(\theta') > \beta_{DA}(\theta)$ a type-θ bidder evaluates the expected utility of the final allocation with respect to the reference point generated by her current beliefs. Denote by $U_{DA}(\tilde{\theta}|\tilde{\theta},\theta';\theta)$ the expected payoff in the Dutch auction of a type-θ bidder who plans to stop the clock at price $\beta_{DA}(\tilde{\theta})$ when the current price is $\beta_{DA}(\theta')$; this reads as

$$\begin{aligned} U_{DA}(\tilde{\theta}|\tilde{\theta},\theta';\theta) \quad = \quad & F_1(\tilde{\theta}|\theta')\left[\theta - \beta_{DA}(\tilde{\theta})\right] - \eta^g\left(\lambda^g - 1\right) F_1(\tilde{\theta}|\theta')\left[1 - F_1(\tilde{\theta}|\theta')\right]\theta \\ & -\eta^m\left(\lambda^m - 1\right) F_1(\tilde{\theta}|\theta')\left[1 - F_1(\tilde{\theta}|\theta')\right]\beta_{DA}(\tilde{\theta}), \end{aligned} \tag{9}$$

where the first term on the right-hand side of (9) is classical expected material utility, whereas the other two terms represent expected gain-loss utility in the item and money dimensions, respectively. The strategy $\beta_{DA}(\theta)$ is an SCPE for a type-θ bidder if, for any $\theta' > \theta$ and any credible deviation $\tilde{\theta} < \theta'$, the CPE condition $U_{DA}(\theta|\theta, \theta'; \theta) \geq U_{DA}(\tilde{\theta}|\tilde{\theta}, \theta'; \theta)$ holds. In other words, a bidder participating in the Dutch auction does not experience news utility from the arrival of new information, but at each time evaluates her action with respect to her current beliefs over the final allocation. Moreover, the only difference between an SCPE in the Dutch auction and a CPE in the FPA is that in the Dutch the plan must be CPE-optimal among all credible plans as beliefs evolve throughout the auction, whereas in the FPA it must be optimal only for ex-ante beliefs, $F_1(\theta)$. Then, we have the following result:

Proposition 9. *Let $\Lambda^k = \eta^k(\lambda^k - 1) \leq 1$ for $k \in \{m, g\}$. Then, under CPE, the Dutch auction raises more revenue than the FPA.*

Proposition 9 shows that also under CPE the Dutch auction raises more revenue than the FPA. The intuition for this result is, again, the attachment effect. Indeed, under CPE a bidder's expected gain-loss utility is U-shaped in her likelihood of winning, so that bidding more aggressively increases the bidder's expected gain-loss utility only if this likelihood is at least 50%; yet, conditional on meeting this threshold, the larger is her likelihood of winning, the more the bidder is tempted to raise her bid. Thus, bidders still bid more aggressively in the Dutch auction since, at the time of stopping the clock, a bidder's likelihood of winning is 100%.[25]

5 Conclusion

In this paper, we have shown that the strategic and revenue equivalence between the Dutch auction and the sealed-bid FPA no longer holds when bidders are expectations-based loss averse, even in the standard symmetric model with independent private values. In particular, even though both auctions allocate the good to the same bidder, the Dutch auction induces a stronger attachment effect than the FPA, thereby generating a higher revenue. Therefore, with expectations-based reference-dependent preferences, two mechanisms that allocate the prize to the same bidder and have the same pricing rule might still result in different payoffs for both the bidders and the seller, depending on how the allocation is implemented.

More generally, the key insight emerging from our analysis is that a seller can increase his revenue by making expectations-based loss-averse bidders more attached to the good at the time of submitting their bid. Indeed, using this general insight, we can rank the four main standard auction formats as follows: the Dutch auction raises more revenue than the FPA, which is revenue equivalent to the SPA; and the latter two formats yield a higher revenue than the English auction.

[25] The only different prediction of CPE is that the FPA revenue dominates the SPA as the latter exposes bidders to additional risk in the money dimension, thereby pushing their bids down; see Lange and Ratan (2010).

Thus, expectations-based loss aversion provides a novel rationale for the use of descending prices by sellers. In fact, besides actual Dutch auctions, in practice several market negotiations feature descending prices as in, for instance, the real estate market where the asking prices of listed properties decline over time until an offer arrives; similarly, the pricing of tickets for sporting or entertainment events also typically follows a descending path. Descending prices can also be used to resolve financial disputes or for the dissolution of partnerships.[26] In all these scenarios, a buyer's key choice is between obtaining the good for sure by accepting the current price, or waiting for the price to drop lower at the risk of the good being "scooped" by another buyer. The second option can be extremely painful for a loss-averse bidder, especially if the price drops slowly.

Finally, we should point out that while our result that sellers should favor the Dutch auction over other selling mechanisms rests on the assumptions that bidders are expectations-based loss averse, some recent papers have provided new arguments in favor of other auction formats when bidders have classical preferences (e.g., risk neutrality). For instance, Akbarpour and Li (2020) consider the properties of various auction formats when the seller cannot commit to follow the rules of the mechanism he proposes. Their analysis strengthens the equivalence between the Dutch auction and the FPA under classical preferences, as they show that both formats are credible mechanisms. However, they also show that the English auction is the only credible strategy-proof mechanism. In contrast, neither the English auction nor the SPA are strategy-proof mechanisms when bidders are expectations-based loss averse; see von Wangenheim (2021). Hence, we can only use revenue and credibility as criteria with expectations-based loss-averse bidders. Since both the FPA and the Dutch auction are credible but the latter yields a higher revenue, the Dutch auction is still the preferred selling mechanism.

Relatedly, Bergemann *et al.* (2017) analyze the FPA under general information structures encompassing independent private values, pure common values and affiliated values as special cases. For any value distribution, they identify a lower bound on the distribution of winning bids and hence on the seller's revenue, allowing them to perform an informationally robust comparison of alternative auction formats. In particular, they show that the FPA (even without a reserve price) always yields a positive revenue, regardless of the information structure, and hence dominates the SPA.[27] Our analysis is less general since we focus on a fixed and simple information structure; however, we conjecture that our result about the superiority of the Dutch auction to hold also in more general auction environments. Indeed, as shown by Balzer and Rosato (2021), the attachment effect continues to drive the incentives of expectations-based loss-averse bidders even in an environment with common or interdependent values.

[26] Qin and Zhang (2013) experimentally compare clock and sealed-bid auctions to dissolve partnerships. Consistent with our model, they find that subjects bid more aggressively in Dutch auctions than in first-price ones, but less aggressively in English auctions than in second-price ones.

[27] Notice that Bergemann *et al.* (2017) only consider static auction formats in their analysis.

A Proofs

Proof of Proposition 1: See Balzer and Rosato (2021). ■

Proof of Lemma 1: Consider a bidder planning to stay in the auction until $\beta_{DA}(\tilde{\theta})$. At price $\beta_{DA}(x) > \beta_{DA}(\tilde{\theta})$, she expects to win with probability $F_1(\tilde{\theta}|x)$. Suppose the price drops to $\beta_{DA}(x-\Delta) = \beta_{DA}(x) - \varepsilon$. If an opponent stops the clock at price $\beta_{DA}(x-\Delta)$ — an event happening with conditional probability $1 - F_1(x-\Delta|x)$ — the bidder loses the auction in which case her gain-loss utility is

$$-\eta^g \lambda^g F_1(\tilde{\theta}|x)\theta + \eta^m F_1(\tilde{\theta}|x)\beta_{DA}(\tilde{\theta}).$$

With probability $F_1(x-\Delta|x)$ no opponent buys and the probability with which the bidder expects to win increases by $F_1(\tilde{\theta}|x-\Delta) - F_1(\tilde{\theta}|x)$. In this case, her gain-loss utility is

$$\eta^g \left[F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x-\Delta)\right] \theta - \eta^m \lambda^m \left[F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x-\Delta)\right] \beta_{DA}(\tilde{\theta}).$$

Hence, her expected news utility when the price drops from $\beta_{DA}(x)$ to $\beta_{DA}(x-\Delta)$ is

$$\begin{aligned}
\mathbb{E}\left[N(x-\Delta|\theta,x)\right] &= \left[1 - F_1\left(x-\Delta|x\right)\right] F_1\left(\tilde{\theta}|x\right)\left[-\eta^g\lambda^g\theta + \eta^m\beta_{DA}\left(\tilde{\theta}\right)\right] \\
&+ F_1\left(x-\Delta|x\right)\left[F_1\left(\tilde{\theta}|x-\Delta\right) - F_1\left(\tilde{\theta}|x\right)\right]\left[\eta^g\theta - \eta^m\lambda^m\beta_{DA}\left(\tilde{\theta}\right)\right] \\
&= \left[1 - F_1\left(x-\Delta|x\right)\right] F_1\left(\tilde{\theta}|x\right)\left[-\eta^g\lambda^g\theta + \eta^m\beta_{DA}\left(\tilde{\theta}\right)\right] \\
&+ \left[-F_1(x-\Delta|x)F_1(\tilde{\theta}|x) + F_1(\tilde{\theta}|x)\right]\left[\eta^g\theta - \eta^m\lambda^m\beta_{DA}(\tilde{\theta})\right] \\
&= -\left[1 - F_1(x-\Delta|x)\right] F_1(\tilde{\theta}|x)[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_{DA}(\tilde{\theta})].
\end{aligned}$$

When the current clock price is $\beta_{DA}(\theta')$, for $x \in [\tilde{\theta}, \theta']$ price $\beta_{DA}(x)$ is reached with probability $F_1(x|\theta')$. Hence, from the perspective of price $\beta_{DA}(\theta')$, the expected news utility associated with a price drop from $\beta_{DA}(x)$ to $\beta_{DA}(x-\Delta)$ is given by:

$$F_1(x|\theta')\mathbb{E}\left[N(x-\Delta|\theta,x)\right] = -F_1(x|\theta')\left[\frac{F_1(x) - F_1(x-\Delta)}{F_1(x)}\right] F_1(\tilde{\theta}|x)[\eta^g(\lambda^g-1)\theta+\eta^m(\lambda^m-1)\beta_{DA}(\tilde{\theta})]. \quad (10)$$

Total expected news utility is the sum of all these incremental expected gain-loss utility terms for all prices from $\beta_{DA}(\theta')$ to $\beta_{DA}(\tilde{\theta})$. Notice that, since β is continuously increasing, as $\varepsilon \to 0$ we have $\Delta \to 0$ and $\frac{F_1(x)-F_1(x-\Delta)}{\Delta F_1(\theta')} \to f_1(x|\theta')$. Hence, the expected news utility, (10), approaches $-\left[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_{DA}(\tilde{\theta})\right] \int_{\tilde{\theta}}^{\theta'} f_1(x|\theta')F_1(\tilde{\theta}|x)dx$. ■

Proof of Proposition 2: We prove Proposition 2 in three steps. First, using only necessary conditions, we derive a lower bound on the equilibrium bid. Then, we focus on sufficient conditions and show that the lower bound is indeed attainable and thus constitutes a PE. Finally, we show that the PPE is the PE that involves the lowest bid.

Step 1. In a symmetric equilibrium, a type-θ bidder prefers executing her plan of buying at price $\beta_{DA}(\theta)$ over buying at price $\beta_{DA}(\tilde{\theta})$ at any clock price $\beta_{DA}(\theta') > \beta_{DA}(\theta)$ if and only if $\Delta U_{DA}(\tilde{\theta}|\theta,\theta') := F_1(\theta')[U_{DA}(\tilde{\theta}|\theta,\theta') - U_{DA}(\theta|\theta,\theta')] \leq 0$ for all $\theta' \geq \theta$ and all credible deviations $\tilde{\theta} \leq \theta'$. For any upward deviation $\tilde{\theta} \geq \theta$ we have

$$\begin{aligned}\Delta U_{DA}(\tilde{\theta}|\theta,\theta') = \; & (1+\eta^g)\,[F_1(\tilde{\theta}) - F_1(\theta)]\theta + \eta^g(\lambda^g - 1)\theta\left(\int_{\theta}^{\theta'} F_1(\theta|x)f_1(x)dx - \int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx\right) \\ & - (1+\eta^m\lambda^m)\,[F_1(\tilde{\theta})\beta_{DA}(\tilde{\theta}) - F_1(\theta)\beta_{DA}(\theta)] \\ & + \eta^m(\lambda^m - 1)\left(\beta_{DA}(\theta)\int_{\theta}^{\theta'} F_1(\theta|x)f_1(x)dx - \beta_{DA}(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx\right). \qquad (11)\end{aligned}$$

Differentiating $\Delta U_{DA}(\tilde{\theta}|\theta,\theta')$ with respect to $\tilde{\theta}$ yields

$$\begin{aligned}\frac{\partial \Delta U_{DA}(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta}} &= (1+\eta^g\lambda^g)\theta f_1(\tilde{\theta}) - (1+\eta^m)\beta_{DA}(\tilde{\theta})f_1(\tilde{\theta}) - (1+\eta^m\lambda^m)F_1(\tilde{\theta})\beta'_{DA}(\tilde{\theta}) \\ & -\eta^g(\lambda^g - 1)\theta\int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x)f_1(x)dx - \eta^m(\lambda^m - 1)\beta_{DA}(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x)f_1(x)dx \\ & -\eta^m(\lambda^m - 1)\beta'_{DA}(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx. \qquad (12)\end{aligned}$$

In equilibrium, the bidder does not want to deviate upwards locally, i.e. $\lim_{\theta' \searrow \theta} \frac{\partial \Delta U_{DA}(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta}} \leq 0$ for $\tilde{\theta} = \theta'$, which leads to the necessary condition

$$(1+\eta^m\lambda^m)\,F_1(\theta)\beta'_{DA}(\theta) + (1+\eta^m)\,\beta_{DA}(\theta)f_1(\theta) \geq (1+\eta^g\lambda^g)\,f_1(\theta)\theta. \qquad (13)$$

Imposing that (13) holds with equality and solving the resulting differential equation using the initial condition $\beta_{DA}(\underline{\theta})F_1(\underline{\theta}) = 0$ yields a lower bound on any PE bid; call this lower bound $\underline{\beta_{DA}}$. This is expression (5) in the main text.

Step 2. We now show that $\underline{\beta_{DA}}$ satisfies the sufficient conditions for a PE. For upward deviations, note that $\frac{\partial^2 \Delta U_{DA}(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}\partial\theta'} < 0$. Hence, a deviation to $\tilde{\theta} > \theta$ is profitable at price $\underline{\beta_{DA}}(\theta') > \underline{\beta_{DA}}(\tilde{\theta})$ if and only if it is profitable at price $\underline{\beta_{DA}}(\theta') = \underline{\beta_{DA}}(\tilde{\theta})$. But for any $\tilde{\theta} = \theta' > \theta$, we have from (12) that the (right-)derivative is

$$\begin{aligned}\left.\frac{\partial \Delta U_{DA}(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta'} &= (1+\eta^g\lambda^g)\theta f_1(\tilde{\theta}) - (1+\eta^m\lambda^m)\,F_1(\tilde{\theta})\beta'_{DA}(\tilde{\theta}) - (1+\eta^m)\,\beta_{DA}(\tilde{\theta})f_1(\tilde{\theta}) \\ &= (1+\eta^g\lambda^g)(\theta - \tilde{\theta})f_1(\tilde{\theta}) < 0\end{aligned}$$

where the second equality follows since (13) holds with equality for type $\tilde{\theta}$.

Next, we show that the bidding function $\underline{\beta_{DA}}$ is immune to downward deviations. Fix $\tilde{\theta} < \theta < \theta'$ and suppose that when the clock price is $\underline{\beta_{DA}}(\theta')$ a type-θ bidder deviates to the plan of buying at price $\underline{\beta_{DA}}(\tilde{\theta}) < \underline{\beta_{DA}}(\theta)$. Such a deviation is only a concern if it is a credible plan; that is, if the bidder actually carries it through. This, however, is not the case. Indeed, since (13) holds with equality for a type-$\tilde{\theta}$ bidder, such bidder would be indifferent towards a local upward deviation around price $\beta_{DA}(\tilde{\theta})$. As the right-hand side of (12) is strictly increasing in θ, and $\theta > \tilde{\theta}$, a type-θ bidder strictly benefits from such a local upward deviation at $\underline{\beta_{DA}}(\tilde{\theta})$.

Step 3. In Step 1 we showed that $\underline{\beta_{DA}}$ is the lowest PE bid. Moreover, notice that all other strictly increasing PE bidding functions that arise in a symmetric equilibrium lead to the same allocation of the good. In equilibrium, no bidder deviates from her strategy and therefore, using (3), it is easy to see that a bidder's equilibrium payoff decreases in her bid. Thus, $\underline{\beta_{DA}}$ is every bidder type's preferred symmetric PE bidding function and hence the PPE. ■

Proof of Proposition 3: In the FPA, the equilibrium utility of bidder with type θ is

$$F_1(\theta)\left[\theta - \beta_{FPA}(\theta)\right] - \left[\eta^g\left(\lambda^g - 1\right)\theta + \eta^m\left(\lambda^m - 1\right)\beta_{FPA}(\theta)\right]F_1(\theta)\left[1 - F_1(\theta)\right].$$

In the Dutch auction, the equilibrium utility of bidder with type θ is

$$F_1(\theta)[\theta - \beta_{DA}(\theta)] - [\eta^g(\lambda^g - 1)\theta + \eta^m(\lambda^m - 1)\beta_{DA}(\theta)]\int_\theta^{\overline{\theta}} f_1(x)F_1(\theta|x)dx.$$

Suppose that $\beta_{FPA}(\theta) = \beta_{DA}(\theta)$. The result then follows since $\int_\theta^{\overline{\theta}} f_1(x)F_1(\theta|x)dx > F_1(\theta)\left[1 - F_1(\theta)\right]$ for $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. ■

Proof of Proposition 4: Recall that $\Delta U_{DA}(\tilde{\theta}|\theta,\theta') := F_1(\theta')[U_{DA}(\tilde{\theta}|\theta,\theta') - U_{DA}(\theta|\theta,\theta')]$. Hence, for any downward deviation $\tilde{\theta} < \theta$ we have

$$\begin{aligned}
&\Delta U_{DA}(\tilde{\theta}|\theta,\theta') = (1+\eta^g\lambda^g)\left[F_1(\tilde{\theta}) - F_1(\theta)\right]\theta + \eta^g(\lambda^g - 1)\theta\left(\int_\theta^{\theta'} F_1(\theta|x)f_1(x)dx - \int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx\right) \\
&-(1+\eta^m)\left[F_1(\tilde{\theta})\beta_{DA}(\tilde{\theta}) - F_1(\theta)\beta_{DA}(\theta)\right] \\
&+\eta^m(\lambda^m - 1)\left(\beta_{DA}(\theta)\int_\theta^{\theta'} F_1(\theta|x)f_1(x)dx - \beta_{DA}(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx\right). \qquad (14)
\end{aligned}$$

Notice that the above expression differs from (11) only for its first and third gain-loss utility terms. Differentiating (14) with respect to $\tilde{\theta}$ yields

$$\frac{\partial \Delta U_{DA}(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta}} = (1+\eta^g\lambda^g+\eta^g(\lambda^g-1))\theta f_1(\tilde{\theta}) - (1+\eta^m-\eta^m(\lambda^m-1))\beta_{DA}(\tilde{\theta})f_1(\tilde{\theta})$$
$$-(1+\eta^m)F_1(\tilde{\theta})\beta'_{DA}(\tilde{\theta}) - \eta^g(\lambda^g-1)\theta\int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x)f_1(x)dx$$
$$-\eta^m(\lambda^m-1)\beta_{DA}(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x)f_1(x)dx - \eta^m(\lambda^m-1)\beta'_{DA}(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx. \qquad (15)$$

As in the PPE condition (13) binds, it follows that for a local downward deviation at the beginning of the auction

$$\left.\frac{\partial \Delta U_{DA}(\tilde{\theta}|\theta,\overline{\theta})}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} = \eta^g(\lambda^g-1)\theta f_1(\theta) + \eta^m(\lambda^m-1)\beta_{DA}(\theta)f_1(\theta)$$
$$+\eta^m(\lambda^m-1)F_1(\theta)\beta'_{DA}(\theta) - \eta^g(\lambda^g-1)\theta\int_{\theta}^{\overline{\theta}} f_1(\theta|x)f_1(x)dx$$
$$-\eta^m(\lambda^m-1)\beta_{DA}(\theta)\int_{\theta}^{\overline{\theta}} f_1(\theta|x)f_1(x)dx - \eta^m(\lambda^m-1)\beta'_{DA}(\theta)\int_{\theta}^{\overline{\theta}} F_1(\theta|x)f_1(x)dx.$$

This expression is positive for $\theta = \overline{\theta}$. For $\theta = \underline{\theta} > 0$ we have

$$\begin{aligned}
& \lim_{\theta\to\underline{\theta}} \left.\frac{\partial \Delta U_{DA}(\tilde{\theta}|\theta,\overline{\theta})}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} \\
= & \lim_{\theta\to\underline{\theta}} f_1(\theta)\left\{\eta^g(\lambda^g-1)\left[1+\ln\left(F_1(\theta)\right)\right]\theta + \eta^m(\lambda^m-1)\left[1+\ln\left(F_1(\theta)\right)\right]\beta_{DA}(\theta)\right\} \\
& + \lim_{\theta\to\underline{\theta}} \eta^m(\lambda^m-1)\beta'_{DA}(\theta)\left[F_1(\theta) - \int_{\theta}^{\overline{\theta}} F_1(\theta|x)f_1(x)dx\right] \\
= & -\infty,
\end{aligned}$$

whereas for $\theta = \underline{\theta} = 0$ we have

$$\frac{\partial \Delta U_{DA}(\tilde{\theta}|\underline{\theta},\overline{\theta})}{\partial \tilde{\theta}}|_{\tilde{\theta}=\underline{\theta}} = -\eta^m(\lambda^m-1)\beta'_{DA}(\underline{\theta})\int_{\theta}^{\overline{\theta}} F_1(\theta|x)f_1(x)dx < 0.$$

Therefore, by continuity, there exists a $\theta^* \in (\underline{\theta},\overline{\theta})$ such that for a local downward deviation, $\frac{\partial \Delta U_{DA}(\tilde{\theta}|\theta,\overline{\theta})}{\partial \tilde{\theta}}|_{\tilde{\theta}=\theta} < 0$ for all $\theta < \theta^*$. Hence, the stated result follows. ■

Proof of Proposition 5: We first show that $\beta_{FPA}(\underline{\theta}) \leq \beta_{DA}(\underline{\theta})$ and with strict inequality if $\underline{\theta} > 0$. Applying integration by parts to $\beta_{FPA}(\theta)$ and $\beta_{DA}(\theta)$, it is easy to see that $\beta_{FPA}(\underline{\theta}) = \frac{1+\eta^g}{1+\eta^m\lambda^m}\underline{\theta}$ and $\beta_{DA}(\underline{\theta}) = \frac{1+\eta^g\lambda^g}{1+\eta^m}\underline{\theta}$. Thus, the claim follows.

Next, observe that the bid in the FPA, (2), is bounded from above by

$\int_{\underline{\theta}}^{\theta} \frac{1+\eta^g\lambda^g}{F_1(\theta)[1+\eta^m\lambda^m]} e^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}[F_1(\theta)-F_1(s)]} f_1(s)sds$. Thus, it is sufficient to show that

$$\left[\frac{F_1(\theta)}{F_1(s)}\right]^{\frac{\eta^m(\lambda^m-1)}{1+\lambda^m\eta^m}} \geq e^{\frac{\eta^m(\lambda^m-1)[F_1(\theta)-F_1(s)]}{1+\eta^m\lambda^m}}$$

which is equivalent to

$$\begin{aligned} & \ln(F_1(\theta)) - \ln(F_1(s)) \geq F_1(\theta) - F_1(s) \\ \Leftrightarrow\; & \ln(F_1(\theta)) - F_1(\theta) \geq \ln(F_1(s)) - F_1(s). \end{aligned} \tag{16}$$

As $\theta \geq s$, (16) holds if $\ln(F_1(x)) - F_1(x)$ is increasing in x. This is the case since $\frac{f_1(x)}{F_1(x)} - f_1(x) = f_1(x)\frac{1-F_1(x)}{F_1(x)} \geq 0$. ■

Proof of Proposition 6: In the Dutch auction, for $x < \theta$ we have that $\left[\frac{F_1(\theta)}{F_1(x)}\right]^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}} > 1$. When gain-loss utility is the same in both dimensions, this implies that

$$\beta_{DA}(\theta) > \int_{\underline{\theta}}^{\theta} \frac{1+\eta^g\lambda^g}{F_1(\theta)\,(1+\eta^m\lambda^m)} x f_1(x)dx = \int_{\underline{\theta}}^{\theta} \frac{x f_1(x)}{F_1(\theta)}dx. \tag{17}$$

Hence, we have overbidding in the Dutch auction.

For the FPA, notice first that

$$\lim_{\theta\to\underline{\theta}} \beta_{FPA}(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1+\eta}{F_1(\theta)\,(1+\eta\lambda)} x f_1(x)dx.$$

Hence, we have underbidding for the lowest type.

Next, we derive a condition on the slope of $\beta_{FPA}(\theta)$. For β_{FPA} to be an equilibrium, the first-order necessary condition of (1) for a local upward deviation yields

$$\begin{aligned} \left.\frac{\partial U(\tilde{\theta}|\theta)}{\partial\tilde{\theta}}\right|_{\tilde{\theta}=\theta} &= f_1(\theta)\left\{1+\eta^g\lambda^g F_1(\theta) + \eta^g\left[1-F_1(\theta)\right]\right\}\theta \\ &\quad - f_1(\theta)\left\{1+\eta^m F_1(\theta) + \eta^m\lambda^m\left[1-F_1(\theta)\right]\right\}\beta_{FPA}(\theta) \\ &\quad - \beta'_{FPA}(\theta)F_1(\theta)\left\{1+\eta^m\lambda^m\left[1-F_1(\theta)\right] + \eta^m\lambda^m F_1(\theta)\right\} \\ &\leq 0. \end{aligned}$$

This inequality can be re-arranged as follows:

$$\beta'_{FPA}(\theta) \leq -\frac{f_1(\theta)}{F_1(\theta)}\left\{\frac{1+\eta^m F_1(\theta)+\eta^m\lambda^m\left[1-F_1(\theta)\right]}{1+\eta^m\lambda^m}\right\}\beta_{FPA}(\theta) + \frac{f_1(\theta)}{F_1(\theta)}\left\{\frac{1+\eta^g\lambda^g F_1(\theta)+\eta^g\left[1-F_1(\theta)\right]}{1+\eta^m\lambda^m}\right\}\theta.$$

Balzer and Rosato (2021) show that the PPE in (2) is obtained as the solution to the differential equation when the above inequality binds. When gain-loss utility is the same in both dimensions, this equation becomes

$$\beta'_{FPA}(\theta) = -\frac{f_1(\theta)}{F_1(\theta)} \left\{ \frac{1+\eta F_1(\theta)+\eta\lambda\left[1-F_1(\theta)\right]}{1+\eta\lambda} \right\} \beta_{FPA}(\theta) + \frac{f_1(\theta)}{F_1(\theta)} \left\{ \frac{1+\eta\lambda F_1(\theta)+\eta\left[1-F_1(\theta)\right]}{1+\eta\lambda} \right\} \theta.$$

Call $\beta^{rn}_{FPA}(\theta)$ the equilibrium bidding function for the risk-neutral benchmark. If $\beta_{FPA}(\theta) < \beta^{rn}_{FPA}$ for all types, then the claim is trivially satisfied for $p = 1$. Otherwise, there exists a θ where β_{FPA} crosses β^{rn}_{FPA} from below; i.e., $\beta_{FPA}(\theta) = \beta^{rn}_{FPA}(\theta)$ and $\beta'_{FPA}(\theta) > (\beta^{rn}_{FPA})'(\theta)$. This implies

$$\frac{\eta(\lambda-1)\left\{F_1(\theta)\beta^{rn}_{FPA}(\theta) - \left[1-F_1(\theta)\right]\theta\right\}}{1+\eta\lambda} > 0. \tag{18}$$

Since $\beta^{rn}_{FPA}(\theta) < \theta$ it follows that $F_1(\theta) = F(\theta)^{N-1} > 0.5$. Hence, at least all types with $F(\theta) < (0.5)^{\frac{1}{N-1}}$ underbid. Moreover, since the left-hand side in (18) is increasing in θ, we have that $\beta'_{FPA}(\theta_0) > (\beta^{rn}_{FPA})'(\theta_0)$ for all $\theta_0 > \theta$. Hence, all types above θ_0 overbid. ■

Proof of Corollary 2: The inequality $Dutch > FPA$ follows from Corollary 1. The equality $FPA = SPA$ follows from the results in Balzer and Rosato (2021). Finally, the inequality $SPA > English$ follows from von Wangenheim (2021). ■

In the proof of Proposition 7, we use the following ancillary result.

Lemma 2. *Let $\alpha \in [0, 1]$ be the weight on a bidder's most recent beliefs in her reference point as the Dutch auction unfolds. Then, for $\theta' > \theta$ the expected utility at price $\beta_{DA}(\theta')$ of a type-θ bidder from an upward deviation from $\beta_{DA}(\theta)$ to $\beta_{DA}(\tilde{\theta})$ is given by*

$$U_\alpha(\tilde{\theta}|\theta,\theta') = \alpha U_{DA}(\tilde{\theta}|\theta,\theta') + (1-\alpha)U_{Sticky}(\tilde{\theta}|\theta,\theta'), \tag{19}$$

where

$$\begin{aligned} U_{Sticky}(\tilde{\theta}|\theta,\theta') \;\; = \;\; & F_1(\tilde{\theta}|\theta')\left[\theta-\beta_{DA}(\tilde{\theta})\right] - \eta^g\lambda^g\left[1-F_1(\tilde{\theta}|\theta')\right]F_1(\theta)\theta + \eta^g F_1(\tilde{\theta}|\theta')\left[1-F_1(\theta)\right]\theta \\ & +\eta^m\left[1-F_1(\tilde{\theta}|\theta')\right]F_1(\theta)\beta_{DA}(\theta) - \eta^m\lambda^m F_1(\tilde{\theta}|\theta')\left[1-F_1(\theta)\right]\beta_{DA}(\tilde{\theta}) \\ & -\eta^m\lambda^m F_1(\tilde{\theta}|\theta')F_1(\theta)\left[\beta_{DA}(\tilde{\theta})-\beta_{DA}(\theta)\right]. \end{aligned} \tag{20}$$

Notice also that U_{Sticky} coincides with the expected utility in (1) for an upward deviation in the FPA after replacing the ex-ante probability $F_1(\tilde{\theta})$ to win with a bid of $\beta_{DA}(\tilde{\theta})$ with the updated probability to win $F_1(\tilde{\theta}|\theta')$ at a given price $\beta_{DA}(\theta')$.

Proof of Lemma 2: We separately analyze the three sources of gain-loss utility:

(i) Under strategy $\beta_{DA}(\theta)$, at price $\beta_{DA}(\theta')$ the winning probability is $F_1(\theta|\theta')$; hence the reference point puts a weight of $(1-\alpha)F_1(\theta)+\alpha F_1(\theta|\theta')$ on winning. A deviation to $\beta_{DA}(\tilde{\theta})$ updates the reference point to $(1-\alpha)F_1(\theta)+\alpha F_1(\tilde{\theta}|\theta')$ and hence induces gain-loss utility

$$\alpha\left\{\eta^g\theta\left[F_1(\tilde{\theta}|\theta')-F_1\left(\theta|\theta'\right)\right]-\eta^m\lambda^m[F_1\left(\tilde{\theta}|\theta'\right)\beta_{DA}(\tilde{\theta})-F_1\left(\theta|\theta'\right)\beta_{DA}(\theta)]\right\}. \tag{21}$$

(ii) Next, we look at expected news utility $\mathbb{E}[N_\alpha(\tilde{\theta}|\theta,\theta')]$ from updates of the reference point that take place during the auction. As in the proof of Lemma 1, consider a price drop from $\beta_{DA}(x)$ to $\beta_{DA}(x-\Delta)$. With probability of $1-F_1(x-\Delta|x)$ an opponent stops the clock; in this case, the bidder loses the auction and experiences gain-loss utility equal to

$$(1-\alpha)F_1(\theta)\left[\eta^m\beta_{DA}(\theta)-\eta^g\lambda^g\theta\right]+\alpha F_1(\tilde{\theta}|x)\left[\eta^m\beta_{DA}(\tilde{\theta})-\eta^g\lambda^g\theta\right]. \tag{22}$$

With probability $F_1(x-\Delta|x)$, no opponent stops the clock, and the bidder updates her belief about winning to $F_1(\tilde{\theta}|x-\Delta)$ and experiences gain-loss utility equal to

$$\alpha\left[F_1(\tilde{\theta}|x-\Delta)-F_1(\tilde{\theta}|x)\right]\left[\eta^g\theta-\eta^m\lambda^m\beta_{DA}(\tilde{\theta})\right]. \tag{23}$$

Notice that combining expression (23) together with the second term in (22) yields exactly α times the expected gain-loss utility from the incremental update in the baseline model for the Dutch auction as calculated in the proof of Lemma 1. Hence, as ε approaches zero, the total expected gain-loss utility from all incremental updates from $\beta_{DA}(\theta')$ to $\beta_{DA}(\tilde{\theta})$ approaches

$$\left[1-F_1(\tilde{\theta}|\theta')\right](1-\alpha)F_1(\theta)\left[\eta^m\beta_{DA}(\theta)-\eta^g\lambda^g\theta\right]+\alpha\mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right]. \tag{24}$$

(iii) With probability $F_1(\tilde{\theta}|\theta')$, the bidder wins the auction at price $\beta_{DA}(\tilde{\theta})$. While her beliefs have updated to winning with certainty at price $\beta_{DA}(\tilde{\theta})$ when she does so, her reference point only fully adjusts when it "sinks in". Comparing the reference point of belief $F_1(\theta)$ to win and pay $\beta_{DA}(\theta)$ with the outcome to win and pay $\beta_{DA}(\tilde{\theta})$ hence induces, from the perspective at price $\beta_{DA}(\theta')$, an expected gain-loss utility of

$$F_1(\tilde{\theta}|\theta')(1-\alpha)\left\{\left[1-F_1(\theta)\right]\eta^g\theta-\left[1-F_1(\theta)\right]\eta^m\lambda^m\beta_{DA}(\tilde{\theta})-F_1(\theta)\eta^m\lambda^m\left[\beta_{DA}(\tilde{\theta})-\beta_{DA}(\theta)\right]\right\}. \tag{25}$$

Finally, by putting all three sources of gain-loss utility together with classical material utility $(1-\alpha)F_1(\tilde{\theta}|\theta')\left[\theta-\beta_{DA}(\tilde{\theta})\right]+\alpha F_1(\tilde{\theta}|\theta')\left[\theta-\beta_{DA}(\tilde{\theta})\right]$, we obtain the formula for $U_{Sticky}(\tilde{\theta}|\theta,\theta')$ as in (19). ■

Equipped with the above result, we are now ready to prove Proposition 7.

Proof of Proposition 7: As in the proof of Proposition 2, we define

$$\Delta U_\alpha(\tilde{\theta}|\theta,\theta') := F_1(\theta')\left[U_\alpha(\tilde{\theta}|\theta,\theta') - U_\alpha(\theta|\theta,\theta')\right].$$

In a symmetric equilibrium, a type-θ bidder prefers executing her plan of buying at price $\beta_{DA}(\theta)$ over buying at price $\beta_{DA}(\tilde{\theta})$ at any clock price $\beta_{DA}(\theta') > \beta_{DA}(\theta)$ if and only if $\Delta U_\alpha(\tilde{\theta}|\theta,\theta') \leq 0$ for all $\theta' \geq \theta$ and all credible deviations $\tilde{\theta} \leq \theta'$. By equations (19) and (20), for any such upward deviations, we have

$$\begin{aligned}\Delta U_\alpha(\tilde{\theta}|\theta,\theta') &= \alpha\Delta U(\tilde{\theta}|\theta,\theta')\\ &\quad + (1-\alpha)\{(1+\eta^g)\,[F_1(\tilde{\theta}) - F_1(\theta)]\theta + \eta^g(\lambda^g-1)\theta F_1(\theta)[F_1(\tilde{\theta}) - F_1(\theta)] \qquad (26)\\ &\quad - (1+\eta^m\lambda^m[1-F_1(\theta)])\,[F_1(\tilde{\theta})\beta_{DA}(\tilde{\theta}) - F_1(\theta)\beta_{DA}(\theta)]\\ &\quad - \eta^m F_1(\theta)\beta_{DA}(\theta)[F_1(\tilde{\theta}) - F_1(\theta)] - \eta^m\lambda^m F_1(\tilde{\theta})F_1(\theta)[\beta_{DA}(\tilde{\theta}) - \beta_{DA}(\theta)]\}. \qquad (27)\end{aligned}$$

Differentiation with respect to $\tilde{\theta}$ yields

$$\begin{aligned}&\frac{\partial\Delta U_\alpha(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}} = \alpha\frac{\partial\Delta U(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}}\\ &+(1-\alpha)\{[1+\eta^g+\eta^g(\lambda^g-1)F_1(\theta)]\theta f_1(\tilde{\theta}) - (1+\eta^m\lambda^m)f_1(\tilde{\theta})\beta_{DA}(\tilde{\theta})\\ &-(1+\eta^m\lambda^m)F_1(\tilde{\theta})\beta'_{DA}(\tilde{\theta}) + \eta^m(\lambda^m-1)F_1(\theta)\beta_{DA}(\theta)f_1(\tilde{\theta})\}. \qquad (28)\end{aligned}$$

Exploiting that the integrals in equation (12) vanish for $\theta' = \theta$, we obtain the following necessary condition for equilibrium:

$$\begin{aligned}0 &\geq \lim_{\theta'\searrow\theta}\frac{\partial\Delta U_\alpha(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}}\Big|_{\tilde{\theta}=\theta'}\\ &= [1+\eta^g+\eta^g(\lambda^g-1)F_1(\theta)]\theta f_1(\theta)\\ &\quad - (1+\eta^m\lambda^m)\beta'_{DA}(\theta)\,F_1(\theta) - [1+\eta^m\lambda^m - \eta^m(\lambda^m-1)F_1(\theta)]\,\beta_{DA}(\theta)f_1(\theta)\\ &\quad + \alpha\,[1-F_1(\theta)]\,[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_{DA}(\theta)]\,f_1(\theta). \qquad (29)\end{aligned}$$

An argument analogous to the one in the proof of Proposition 2 shows that the solution to the differential equation obtained by making condition (29) bind is the PPE for the Dutch auction. It is easy to verify that the slope of this differential equation

$$\beta'_{DA}(\theta) = \frac{[1+\eta^g+\eta^g(\lambda^g-1)F_1(\theta)]\theta f_1(\theta) - [1+\eta^m\lambda^m-\eta^m(\lambda^m-1)F_1(\theta)]\,\beta_{DA}(\theta)f_1(\theta)}{(1+\eta^m\lambda^m)\,F_1(\theta)} + \alpha\frac{[1-F_1(\theta)]\,[\eta^g(\lambda^g-1)\theta+\eta^m(\lambda^m-1)\beta_{DA}(\theta)]\,f_1(\theta)}{(1+\eta^m\lambda^m)\,F_1(\theta)}$$

is increasing in α. Hence, given the same initial condition $\beta_{DA}(\underline{\theta})F_1(\underline{\theta}) = 0$ it follows that $\beta_{DA}(\theta)$ is increasing in α for all $\theta > \underline{\theta}$. ■

Proof of Proposition 8: For $\tilde{\theta} \neq \theta$ we have

$$\begin{aligned}
\frac{\partial U_{FPA}(\tilde{\theta}|\theta)}{\partial\tilde{\theta}} =& f_1(\tilde{\theta})\left\{\theta - \beta_{FPA}(\tilde{\theta}) + (\eta^g + \Lambda^g_{\tilde{\theta}<\theta})\theta - (\eta^m\lambda^m - \Lambda^m_{\tilde{\theta}<\theta})\beta_{FPA}(\theta) - (\eta^m\lambda^m - \Lambda^m_{\tilde{\theta}<\theta})\left[\beta_{FPA}(\tilde{\theta}) - \beta_{FPA}(\theta)\right]\right\} \\
&- (1-2F_1(\tilde{\theta}))f_1(\tilde{\theta})\left[(\eta^g\lambda^g - \eta^g)\theta + (\eta^m\lambda^m - \eta^m)\beta_{FPA}(\tilde{\theta})\right] \\
&- (1+\eta^m\lambda^m - \Lambda^m_{\tilde{\theta}<\theta})\beta'_{FPA}(\tilde{\theta})F_1(\tilde{\theta}) - (\eta^m\lambda^m - \eta^m)(1-F_1(\tilde{\theta}))F_1(\tilde{\theta})\beta'_{FPA}(\tilde{\theta}) \\
=& f_1(\tilde{\theta})\left[1 + \eta^g + \Lambda^g_{\tilde{\theta}<\theta} - (1-2F_1(\tilde{\theta}))(\eta^g\lambda^g - \eta^g)\right]\theta \\
&- f_1(\tilde{\theta})\left[1 + \eta^m\lambda^m - \Lambda^m_{\tilde{\theta}<\theta} + (1-2F_1(\tilde{\theta}))(\eta^m\lambda^m - \eta^m)\right]\beta_{FPA}(\tilde{\theta}) \\
&- F_1(\tilde{\theta})\left[1 + \eta^m\lambda^m - \Lambda^m_{\tilde{\theta}<\theta} + (1-F_1(\tilde{\theta}))(\eta^m\lambda^m - \eta^m)\right]\beta'_{FPA}(\tilde{\theta})
\end{aligned}$$

The local (necessary) conditions for an equilibrium β_{FPA} are satisfied if the left-derivative and right-derivative at $\tilde{\theta} = \theta$ satisfy $\frac{\partial_- U_{FPA}}{\partial\tilde{\theta}} \geq 0 \geq \frac{\partial_+ U_{FPA}}{\partial\tilde{\theta}}$, which is equivalent to

$$\beta'_{FPA}(\theta) \leq \frac{1+\eta^g\lambda^g - (1-2F_1(\theta))(\eta^g\lambda^g - \eta^g)}{1+\eta^m + (1-F_1(\theta))(\eta^m\lambda^m - \eta^m)}\frac{\theta f_1(\theta)}{F_1(\theta)} - \frac{1+\eta^m + (1-2F_1(\theta))(\eta^m\lambda^m - \eta^m)}{1+\eta^m + (1-F_1(\theta))(\eta^m\lambda^m - \eta^m)}\frac{\beta_{FPA}(\theta)f_1(\theta)}{F_1(\theta)} \tag{30}$$

and

$$\beta'_{FPA}(\theta) \geq \frac{1+\eta^g - (1-2F_1(\theta))(\eta^g\lambda^g - \eta^g)}{1+\eta^m\lambda^m + (1-F_1(\theta))(\eta^m\lambda^m - \eta^m)}\frac{\theta f_1(\theta)}{F_1(\theta)} - \frac{1+\eta^m\lambda^m + (1-2F_1(\theta))(\eta^m\lambda^m - \eta^m)}{1+\eta^m\lambda^m + (1-F_1(\theta))(\eta^m\lambda^m - \eta^m)}\frac{\beta_{FPA}(\theta)f_1(\theta)}{F_1(\theta)}. \tag{31}$$

Note that for given values of θ and $\beta_{FPA}(\theta)$ indeed the right-hand side of (30) is larger than the right-hand side of (31). Hence, the lowest PE candidate that satisfies the local necessary conditions is given by the solution of the differential equation where (31) binds with equality, together with initial condition $\beta_{FPA}(\underline{\theta})F_1(\underline{\theta}) = 0$.[28] For sufficiency, note that single crossing is satisfied; i.e., for

[28] Deriving the closed-form solution of this differential equation is straightforward, yet unnecessary for the argument of the proof.

$\tilde{\theta} \neq \theta$ we have

$$\frac{\partial^2 U_{FPA}(\tilde{\theta}|\theta)}{\partial\tilde{\theta}\partial\theta} = 1 + \eta^g + \Lambda^g_{\tilde{\theta}<\theta} - (1 - 2F_1(\tilde{\theta}))(\eta^g\lambda^g - \eta^g) \geq 1 + \eta^g - (\eta^g\lambda^g - \eta^g) > \eta^g \geq 0,$$

where the last inequality follows from $\Lambda^g \leq 1$. Hence, the solution is a PE, and, again, since it is the lowest among all potential symmetric PE it is the PPE.

We now compare this PPE to the one obtained with the utility specification (1) in the main text. The derivative for (1) with respect to $\tilde{\theta}$ reads

$$\begin{aligned}\frac{\partial U_{FPA}(\tilde{\theta}|\theta)}{\partial\tilde{\theta}} =& f_1(\tilde{\theta})\Big[1 + \eta^g\lambda^g F_1(\theta) + \eta^g(1 - F_1(\theta))\Big]\theta \\ &- f_1(\tilde{\theta})\Big[\Big(1 + \eta^m\lambda^m(1 - F_1(\theta))\Big)\beta_{FPA}(\tilde{\theta}) + \eta^m F_1(\theta)\beta_{FPA}(\theta) + \eta^m\lambda^m F_1(\theta)(\beta_{FPA}(\tilde{\theta}) - \beta_{FPA}(\theta))\Big] \\ &- (1 + \eta^m\lambda^m)F_1(\tilde{\theta})\beta'_{FPA}(\tilde{\theta}).\end{aligned}$$

By equating the above expression to zero and setting $\tilde{\theta} = \theta$, the necessary condition for a PE can be written as

$$\beta'_{FPA}(\theta) = \frac{1 + \eta^g\lambda^g F_1(\theta) + \eta^g(1 - F_1(\theta))}{1 + \eta^m\lambda^m}\frac{F_1(\theta)\theta}{F_1(\theta)} - \frac{1 + \eta^m\lambda^m(1 - F_1(\theta)) + \eta^m F_1(\theta)}{1 + \eta^m\lambda^m}\frac{F_1(\theta)\beta_{FPA}(\theta)}{F_1(\theta)}. \quad (32)$$

Balzer and Rosato (2021) show that this differential equation with initial condition $\beta_{FPA}(\underline{\theta})F_1(\underline{\theta}) = 0$ solves to the PPE as described in Proposition 1. The required revenue ranking follows from the fact that the right-hand side in (31) is strictly smaller than the right-hand side in (32). Thus, the solution to the differential equation in (32) is larger than that of (31). ■

Proof of Proposition 9: First, we re-write Equation (9) as

$$U_{DA}(\tilde{\theta}|\tilde{\theta},\theta';\theta) = F_1(\tilde{\theta}|\theta')[\theta - \beta_{DA}(\tilde{\theta})] - \Lambda^g\theta F_1(\tilde{\theta}|\theta')\left[1 - F_1(\tilde{\theta}|\theta')\right] - \Lambda^m\beta_{DA}(\tilde{\theta})F_1(\tilde{\theta}|\theta')[1 - F_1(\tilde{\theta}|\theta')].$$

Notice that this utility function is continuously differentiable in $\tilde{\theta}$ for any $\theta' > \tilde{\theta}$. A necessary condition for a CPE in the FPA is that for a type-θ bidder, $\theta = \tilde{\theta}$ maximizes $U_{DA}(\tilde{\theta}|\tilde{\theta},\theta';\theta)$ at the beginning of the auction, i.e. for $\theta' = \overline{\theta}$. Differentiating $U_{DA}\left(\tilde{\theta}|\tilde{\theta},\overline{\theta};\theta\right)$ with respect to $\tilde{\theta}$ and evaluating the resulting first-order condition at $\tilde{\theta} = \theta$ yields a differential equation whose solution provides us with the equilibrium bidding strategy. The symmetric CPE bidding strategy in the FPA, borrowed from Lange and Ratan (2010), is given by

$$\beta_{FPA}(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1 + \Lambda^g[2F_1(x) - 1]}{F_1(\theta)\left\{1 + \Lambda^m\left[1 - F_1(\theta)\right]\right\}} x f_1(x)dx. \quad (33)$$

For the Dutch auction, strategy $\beta_{DA}(\theta)$ for type-θ is credible if and only if $\theta = \tilde{\theta}$ maximizes $U_{DA}(\tilde{\theta}|\tilde{\theta}, \theta'; \theta)$ for $\theta' \to \theta$. Since

$$\begin{aligned}\frac{\partial U_{DA}(\tilde{\theta}|\tilde{\theta}, \theta'; \theta)}{\partial \tilde{\theta}} &= -F_1(\tilde{\theta}|\theta')\beta'_{DA}(\tilde{\theta})\left\{1 - \Lambda^m\left[1 - F_1(\tilde{\theta}|\theta')\right]\right\} \\ &\quad + f_1(\tilde{\theta}|\theta')\left\{\theta - \beta_{DA}(\tilde{\theta}) - \Lambda^g\theta\left[1 - 2F_1(\tilde{\theta}|\theta')\right] - \Lambda^m\beta_{DA}(\tilde{\theta})\left[1 - 2F_1(\tilde{\theta}|\theta')\right]\right\},\end{aligned}$$

evaluating the necessary condition at $\tilde{\theta} = \theta' = \theta$ yields the differential equation

$$-\beta'_{DA}(\theta) + \frac{f_1(\theta)}{F_1(\theta)}\left[\theta - \beta_{DA}(\theta) + \Lambda^g\theta + \Lambda^m\beta_{DA}(\theta)\right] = 0.$$

Using the initial condition $\beta_{DA}(\underline{\theta})F_1(\underline{\theta}) = 0$, the solution to the above differential equation provides us with the unique equilibrium candidate:

$$\beta_{DA}(\theta) = \frac{1}{F_1(\theta)^{1-\Lambda^m}}\int_{\underline{\theta}}^{\theta}\frac{1+\Lambda^g}{F_1(x)^{\Lambda^m}}xf_1(x)dx. \tag{34}$$

Since the equilibrium candidate is the only time-consistent candidate, it only remains to show sufficiency (i.e., global deviations to $\tilde{\theta} < \theta$ at $\theta = \theta'$). Suppose that when the clock price is $\beta_{DA}(\theta)$, a type-θ bidder deviates to the plan of buying at price $\beta_{DA}(\tilde{\theta}) < \beta_{DA}(\theta)$. Such a deviation is only a concern if it is a credible plan; that is, if the bidder actually carries it through. This, however, is not the case. Indeed, since for $\theta = \tilde{\theta}$ we have $\frac{\partial U_{DA}(\tilde{\theta}|\tilde{\theta},\tilde{\theta};\tilde{\theta})}{\partial \tilde{\theta}} = 0$, a type-$\tilde{\theta}$ bidder would be indifferent towards a local upward deviation around price $\beta_{DA}(\tilde{\theta})$. Since $\Lambda^g \leq 1$, $\frac{\partial U_{DA}(\tilde{\theta}|\tilde{\theta},\theta';\theta)}{\partial \tilde{\theta}\partial\theta} = f_1(\tilde{\theta}|\theta')\left[(1-\Lambda^g) + \Lambda^g 2F_1(\tilde{\theta}|\theta')\right] > 0$ for any θ' and in particular for $\theta' = \tilde{\theta}$; hence, a type-θ bidder strictly benefits from such a local upward deviation at $\beta_{DA}(\tilde{\theta})$. This establishes (34) as the unique SCPE in the Dutch auction.

Finally, in order to establish the revenue ranking, we need to show that

$$\beta_{DA}(\theta) = F_1(\theta)^{\Lambda^m - 1}\int_{\underline{\theta}}^{\theta}\frac{(1+\Lambda^g)\,xf_1(x)}{F_1(x)^{\Lambda^m}}dx \geq \int_{\underline{\theta}}^{\theta}\frac{\left[(1-\Lambda^g) + 2\Lambda^g F_1(x)\right]xf_1(x)}{F_1(\theta)\left\{1 + \Lambda^m\left[1 - F_1(\theta)\right]\right\}}dx = \beta_{FPA}(\theta). \tag{35}$$

To establish (35), it is sufficient to show that

$$F_1(\theta)^{\Lambda^m}\int_{\underline{\theta}}^{\theta}\frac{1}{F_1(x)^{\Lambda^m}}f_1(x)xdx \geq \int_{\underline{\theta}}^{\theta}xf_1(x)dx,$$

which is equivalent to

$$\Leftrightarrow \int_{\underline{\theta}}^{\theta}\frac{xf_1(x)}{F_1(x)^{\Lambda^m}}dx \geq \int_{\underline{\theta}}^{\theta}\frac{xf_1(x)}{F_1(\theta)^{\Lambda^m}}dx.$$

The result then follows since $F_1(x)^{\Lambda^m} < F_1(\theta)^{\Lambda^m}$ for $\theta > x$. ■

References

[1] Abeler, J., A. Falk, L. Goette and D. Huffman (2011), “Reference Points and Effort Provision” *American Economic Review*, 101(2), 470-492.

[2] Akbarpour, M. and S. Li (2020), “Credible Auctions: A Trilemma” *Econometrica*, 88(2), 425-467.

[3] Allais, M. (1953), “Le comportement de l’homme rationnel devant le risque: Critique des postulats et axiomes de l’école Américaine” *Econometrica*, 21(4), 503–546.

[4] Auster, S. and C. Kellner (2022), “Robust Bidding and Revenue in Descending Price Auctions” *Journal of Economic Theory*, 199, 105072.

[5] Balzer, B. and A. Rosato (2021), “Expectations-Based Loss Aversion in Auctions with Interdependent Values: Extensive vs. Intensive Risk” *Management Science*, 67(2), 1056-1074.

[6] Banerji, A. and N. Gupta (2014), “Detection, Identification and Estimation of Loss Aversion: Evidence from an Auction Experiment” *American Economic Journal: Microeconomics*, 6(1), 91-133.

[7] Bergemann, D., B. Brooks and S. Morris (2017), “First-price Auctions with General Information Structures: Implications for Bidding and Revenue” *Econometrica*, 85(1), 107–143.

[8] Carare, O. and M. Rothkopf (2005), “Slow Dutch Auctions” *Management Science*, 51(3), 365-373.

[9] Card, D. and G. Dahl (2011), “Family Violence and Football: The Effect of Unexpected Emotional Cues on Violent Behavior” *Quarterly Journal of Economics*, 126(1), 103–143.

[10] Cerulli-Harms, A., L. Goette and C. Sprenger (2019), “Randomizing Endowments: An Experimental Study of Rational Expectations and Reference-Dependent Preferences” *American Economic Journal: Microeconomics*, 11(1), 185–207.

[11] Cheema, A., D. Chakravarti and A. Sinha (2012), “Bidding Behavior in Descending and Ascending Auctions” *Marketing Science*, 31(5), 779–800.

[12] Coppinger, V., V. Smith and J. Titus (1980), “Incentives and Behavior in English, Dutch and Sealed-bid Auctions” *Economic Inquiry*, 18(1), 1–22.

[13] Cox, J., B. Robertson and V. Smith (1982), “Theory and behavior of single object auctions” in Research in Experimental Economics (Vernon L. Smith, ed.), 1–43, JAI Press, Greenwich, Connecticut.

[14] Cox, J., V. Smith and J. Walker (1983), "A Test that Discriminates between Two Models of the Dutch-First Auction Non-Isomorphism" *Journal of Economic Behavior & Organization*, 4(2-3), 205–219.

[15] Crawford, V. and J.J. Meng (2011), "New York City Cabdrivers' Labor Supply Revisited: Reference-Dependent Preferences with Rational Expectations Targets for Hours and Income" *American Economic Review* 101(5), 1912–1932.

[16] Daido, K. and T. Murooka (2016), "Team Incentives and Reference-Dependent Preferences" *Journal of Economics & Management Strategy*, 25(4), 958-989.

[17] Dato, S., A. Grunewald, D. Müller and P. Strack (2017), "Expectations-Based Loss Aversion and Strategic Interaction" *Games and Economic Behavior*, 104, 681-705.

[18] Dreyfuss, B., O. Heffetz and M. Rabin (2021), "Expectations-Based Loss Aversion May Help Explain Seemingly Dominated Choices in Strategy-Proof Mechanisms" forthcoming in *American Economic Journal: Microeconomics.*

[19] Ehrhart, K.M. and M. Ott (2017), "Loss-averse Bidders in English and Dutch Auctions" *Working Paper.*

[20] Eisenhuth, R. (2019), "Reference Dependent Mechanism Design" *Economic Theory Bulletin*, 7(1), 77-103.

[21] Ericson Marzilli, K.M. and A. Fuster (2011), "Expectations as Endowments: Evidence on Reference-Dependent Preferences from Exchange and Valuation Experiments" *Quarterly Journal of Economics*, 126(4), 1879-1907.

[22] Fugger, N., P. Gillen and T. Riehm (2020), "Procurement design with loss averse bidders" *Working Paper.*

[23] Gill, D. and V. Prowse (2012), "A Structural Analysis of Disappointment Aversion in a Real Effort Competition" *American Economic Review*, 102(1), 469-503.

[24] Gneezy, U., L. Goette, C. Sprenger and F. Zimmermann (2017), "The Limits of Expectations-Based Reference Dependence" *Journal of the European Economic Association*, 15(4), 861-876.

[25] Harstad, R. (2000), "Dominant Strategy Adoption and Bidders' Experience with Pricing Rules" *Experimental Economics*, 3(3), 261–280.

[26] Heffetz, O. and J. List (2014), "Is the Endowment Effect an Expectations Effect?" *Journal of the European Economic Association*, 12(5), 1396–1422.

[27] Heffetz, O. (2021), “Are reference points merely lagged beliefs over probabilities?” *Journal of Economic Behavior & Organization*, 181, 252-269.

[28] Heidhues, P. and B. Kőszegi (2008), “Competition and Price Variation when Consumers are Loss Averse” *American Economic Review*, 98(4), 1245-1268.

[29] Heidhues, P. and B. Kőszegi (2014) “Regular Prices and Sales” *Theoretical Economics*, 9(1), 217–251.

[30] Herweg, F., H. Karle and D. Müller (2018), “Incomplete Contracting, Renegotiation, and Expectation-Based Loss Aversion” *Journal of Economic Behavior & Organization*, 145, 176-201.

[31] Herweg, F. and K. Mierendorff (2013), “Uncertain Demand, Consumer Loss Aversion, and Flat-Rate Tariffs” *Journal of the European Economic Association*, 11(2), 399-432.

[32] Herweg, F., D. Müller and P. Weinschenk (2010), “Binary Payment Schemes: Moral Hazard and Loss Aversion” *American Economic Review*, 100(5), 2451-2477.

[33] Imas, A. (2016), “The realization effect: Risk-taking after Realized versus Paper Losses” *American Economic Review*, 106(8), 2086–2109.

[34] Kagel, J., R. Harstad and D. Levin (1987), “Information Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study” *Econometrica*, 55(6), 1275–1304.

[35] Karle, H., G. Kirchsteiger and M. Peitz (2015), “Loss Aversion and Consumption Choice: Theory and Experimental Evidence” *American Economic Journal: Microeconomics*, 7(2), 101-120.

[36] Karle, H. and M. Peitz (2014), “Competition under Consumer Loss Aversion” *RAND Journal of Economics*, 45(1), 1-31.

[37] Karle, H. and M. Peitz (2017), “De-Targeting: Advertising an Assortment of Products to Loss-Averse Consumers” *European Economic Review*, 95, 103-124.

[38] Karle, H. and H. Schumacher (2017), “Advertising and Attachment: Exploiting Loss Aversion through Pre-Purchase Information” *RAND Journal of Economics*, 48(4), 927-948.

[39] Karni E. (1988) “On the Equivalence between Descending Bid Auctions and First Price Sealed Bid Auctions” *Theory and Decision*, 25(3), 211-217.

[40] Karni E. and Z. Safra (1989) “Dynamic Consistency, Revelations in Auctions and the Structure of Preferences” *Review of Economic Studies*, 56(3), 421-433.

[41] KATOK, E. and A. KWASNICA (2008), "Time is Money: The Effect of Clock Speed on Seller's Revenue in Dutch Auctions" *Experimental Economics*, 11(4), 344-357.

[42] KŐSZEGI, B. and M. RABIN (2006), "A Model of Reference-Dependent Preferences" *Quarterly Journal of Economics*, 121(4), 1133-1165.

[43] KŐSZEGI, B. and M. RABIN (2007), "Reference-Dependent Risk Attitudes" *American Economic Review*, 97(4), 1047-1073.

[44] KŐSZEGI, B. and M. RABIN (2009), "Reference-Dependent Consumption Plans" *American Economic Review*, 99(3), 909-936.

[45] LANGE, A. and A. RATAN (2010), "Multi-dimensional Reference-dependent Preferences in Sealed-bid auctions: How (most) Laboratory Experiments Differ from the Field" *Games and Economic Behavior*, 68(2), 634-645.

[46] LUCKING-REILEY, D. (1999), "Using Field Experiments to Test Equivalence between Auction Formats: Magic on the Internet" *American Economic Review*, 89(5), 1063-1080.

[47] MACERA, R. (2018) "Intertemporal Incentives Under Loss Aversion" *Journal of Economic Theory*, 178, 551-594.

[48] MASATLIOGLU, Y. and C. RAYMOND (2016), "A Behavioral Analysis of Stochastic Reference Dependence" *American Economic Review*, 106(9), 2760-2782.

[49] MEISNER, V. and J. VON WANGENHEIM (2020), "School Choice and Loss Aversion" *Working Paper*.

[50] MYERSON, R. (1979) "Incentive Compatibility and the Bargaining Problem" *Econometrica*, 47(1), 61-73.

[51] MYERSON, R. (1981) "Optimal Auction Design" *Mathematics of Operations Research*, 6(1), 58-73.

[52] NAKAJIMA, D. (2011), "First-price Auctions, Dutch Auctions, and Buy-it-now Prices with Allais Paradox Bidders" *Theoretical Economics*, 6(3), 473-498.

[53] NEILSON, W. (1994), "Second-price Auctions, Dutch Auctions without Expected Utility" *Journal of Economic Theory*, 62(1), 136-151.

[54] PAGEL, M. (2016), "Expectations-Based Reference-Dependent Preferences and Asset Pricing" *Journal of the European Economic Association*, 14(2), 468–514.

[55] PAGEL, M. (2017), "Expectations-Based Reference-Dependent Life-Cycle Consumption" *Review of Economic Studies*, 84(2), 885–934.

[56] Pagel, M. (2018), "A News-Utility Theory for Inattention and Delegation in Portfolio Choice" *Econometrica*, 86(2), 491–522.

[57] Pagel, M. (2019), "Prospective Gain-loss Utility: Ordered versus Separated Comparison" *Journal of Economic Behavior & Organization*, 168, 62–75.

[58] Pope, D. and M. Schweitzer (2011) "Is Tiger Woods Loss Averse? Persistent Bias in the Face of Experience, Competition, and High Stakes" *American Economic Review*, 101(1), 129–157.

[59] Qin, X. and F. Zhang (2013) "Using Clock Auctions to Dissolve Partnership: An Experimental Study" *Economic Letters*, 119, 55–59.

[60] Rosato, A. (2016), "Selling Substitute Goods to Loss-Averse Consumers: Limited Availability, Bargains and Rip-offs" *RAND Journal of Economics*, 47(3), 709-733.

[61] Rosato, A. (2017), "Sequential Negotiations with Loss-Averse Buyers" *European Economic Review*, 91, 290-304.

[62] Rosato, A. (2022), "Loss Aversion in Sequential Auctions" *Working Paper*.

[63] Rosato, A. and A. Tymula (2019), "Loss Aversion and Competition in Vickrey Auctions: Money Ain't No Good" *Games and Economic Behavior*, 115, 188-208.

[64] Smith, A. (2019), "Lagged Beliefs and Reference-Dependent Utility" *Journal of Economic Behavior & Organization*, 167, 331-340.

[65] Sprenger, C. (2015), "An Endowment Effect for Risk: Experimental Tests of Stochastic Reference Points" *Journal of Political Economy*, 123(6), 1456-1499.

[66] Thakral, N. and L. Tô (2021), "Daily Labor Supply and Adaptive Reference Points" *American Economic Review*, 111(8), 2417-2443.

[67] Vickrey, W. (1961), "Counterspeculation and Competitive Sealed Tenders" *Journal of Finance*, 16(1), 8-37.

[68] von Wangenheim, J. (2021), "English vs. Vickrey Auctions with Loss Averse Bidders" *Journal of Economic Theory,* 197, 105328.

[69] Weber, R. (1982), "The Allais paradox, Dutch auctions, and alpha-utility theory" *Working Paper*.

[70] Zimmermann, F. (2015), "Clumped or Piecewise? - Evidence on Preferences for Information" *Management Science*, 61(4), 740–753.